Lattice QCD studies of strangeness S = -2 baryon-baryon interactions

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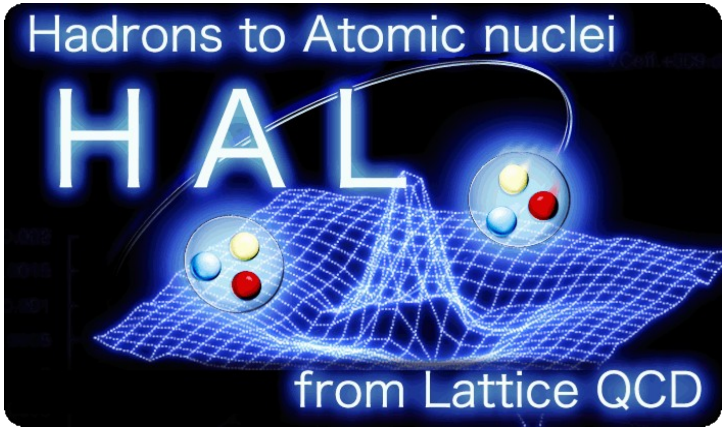
for HAL QCD collaboration



HAL (Hadrons to Atomic nuclei from Lattice) QCD Collaboration

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Introduction

Strangeness in nuclei opened the new frontier of nuclear physics.

Experimental side

Exploration of the multi-strangeness hadronic systems is planned at J-PARC

- Generalized BB interaction
- Hypernuclear structure
- Search of exotic hadrons
- and so on

J-PARC

Theoretical side

Study the hyperon-nucleon (YN) and hyperon-hyperon (YY) interactions

One of the most important subject in the (hyper-) nuclear physics

The phenomenological description of them has large uncertainties due to the shortage of experimental data.

Lattice QCD simulation can produce BB potential directly from QCD complementary to an experiment.

Introduction

This work :

Baryon-baryon interactions in strangeness S = -2 system

- The first step towards the multi-strangeness world.
- Structures of double-Λ hypernuclei and Ξ-hypernuclei
- The SU(3) breaking effects in the BB interaction.
- "H-dibaryon" at physical point .

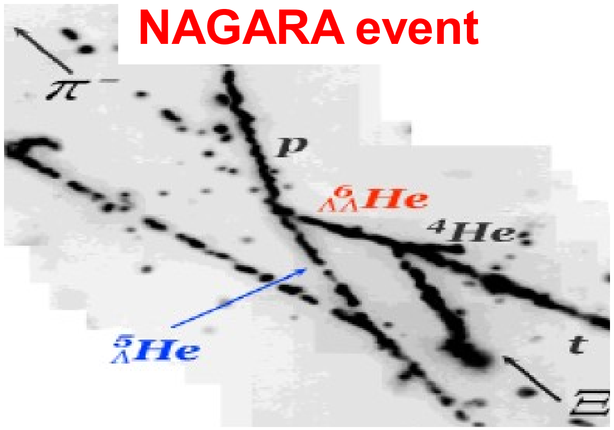
Information of $\Lambda\Lambda$ interaction and H-dibaryon from experiment

Conclusions of the "NAGARA Event" (The double- hypernuclear event)

Lower limit of "H" mass : $m_H \geq 2m_\Lambda$ - 6.9MeV.

The Λ–Λ interaction is weakly attractive.

K.Nakazawa and KEK-E176 & E373 collaborators



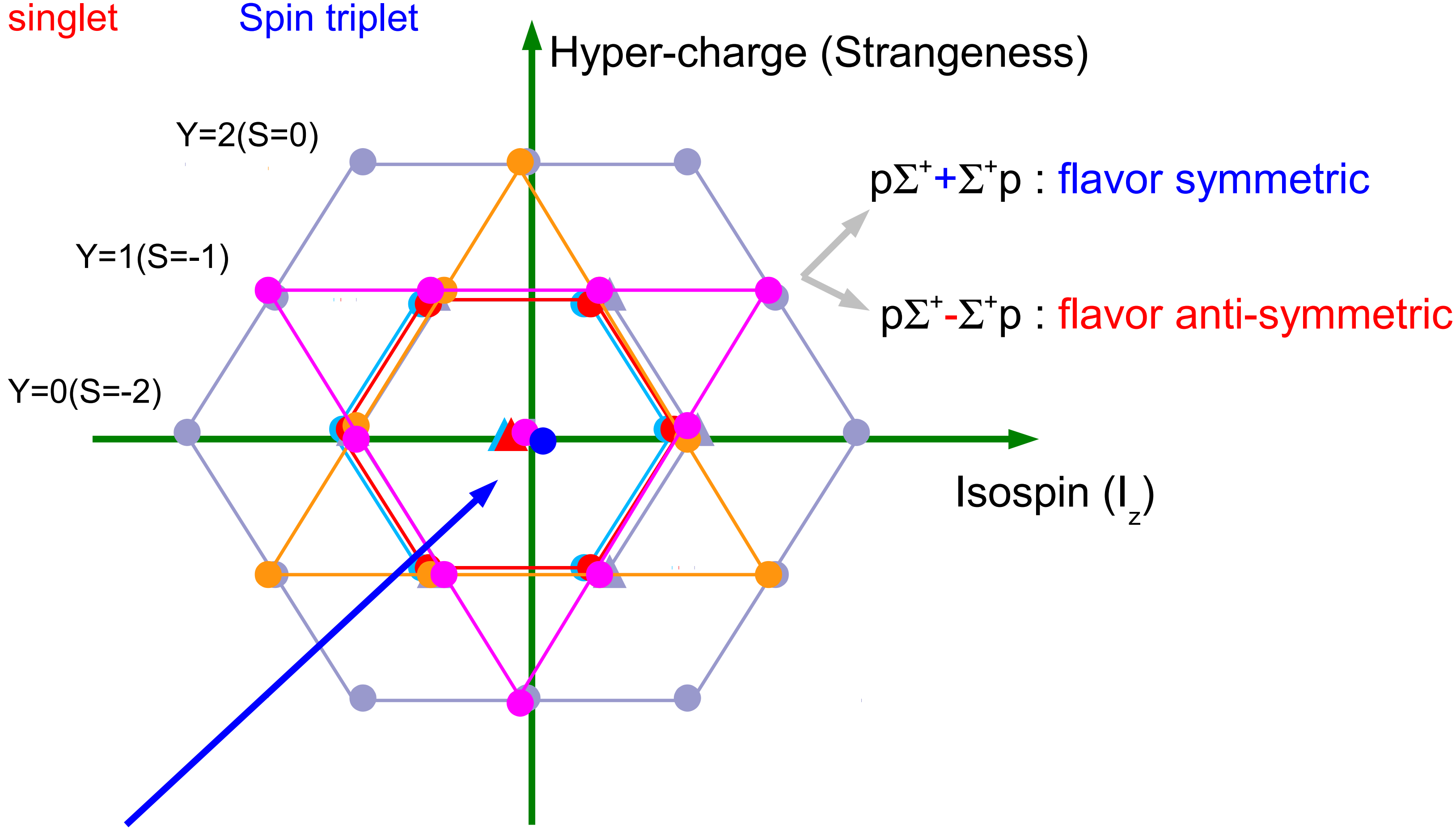
SU(3) Classification of B-B states

Within S-wave total anti-symmetric states are constructed by combination of spin and flavor.

$8 \times 8 = 1 + 8_S + 27 + 8_A + 10 + 10^*$

Flavor symmetric, Spin singlet: $1 + 8_S + 27$

Flavor anti-symmetric, Spin triplet: $8_A + 10 + 10^*$

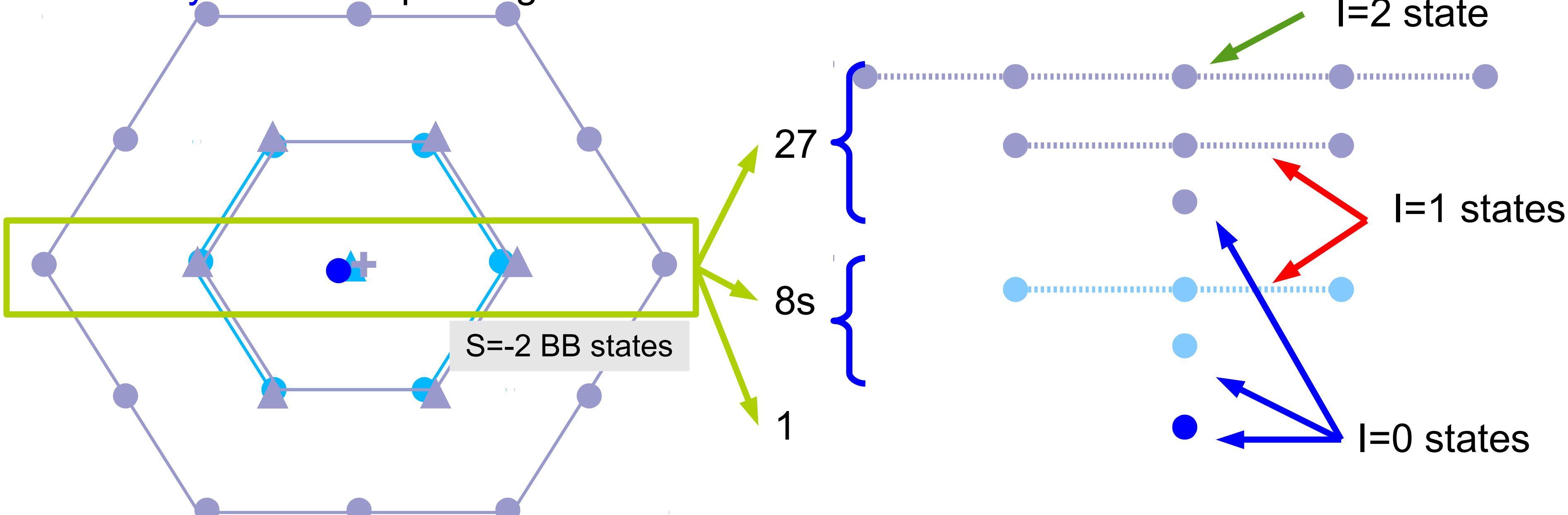


There are 10 flavor combinations

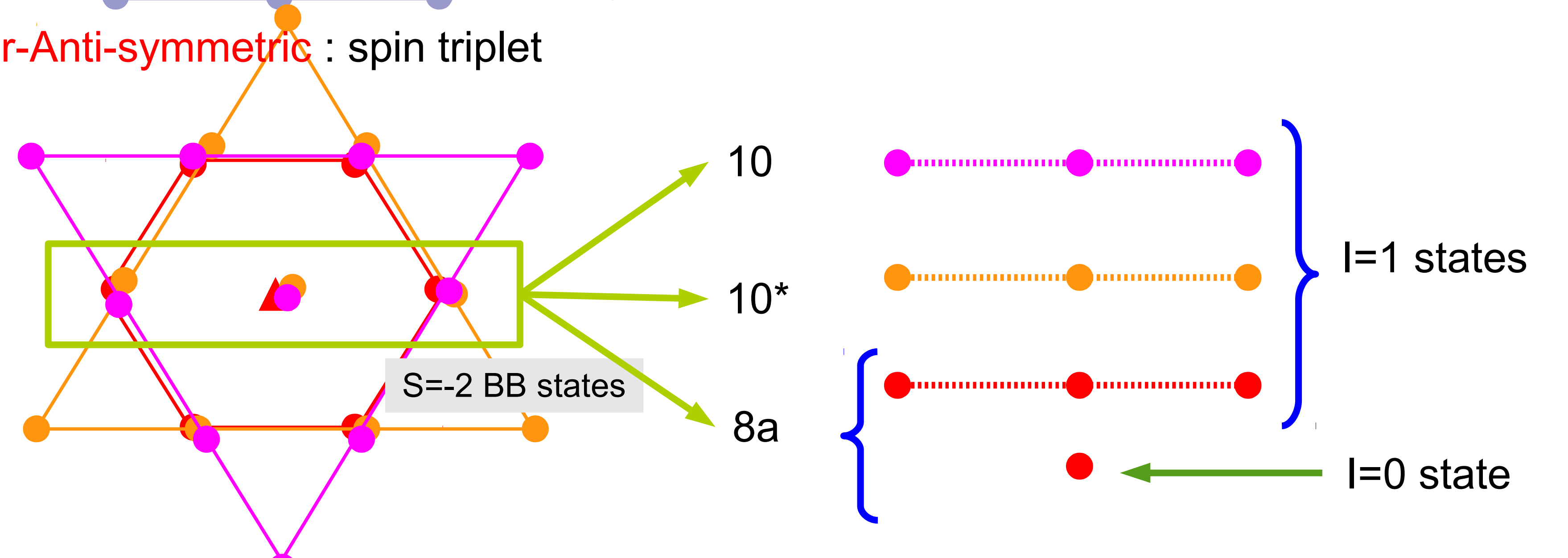
$$\Lambda\Lambda,\ p\Xi^-,\ n\Xi^0,\ \Xi^- p,\ \Xi^0 n,\ \Sigma^+\Sigma^-,\ \Sigma^0\Sigma^0,\ \Sigma^-\Sigma^+,\ \Lambda\Sigma^0,\ \Sigma^0\Lambda$$

Classification of B-B states with S=-2

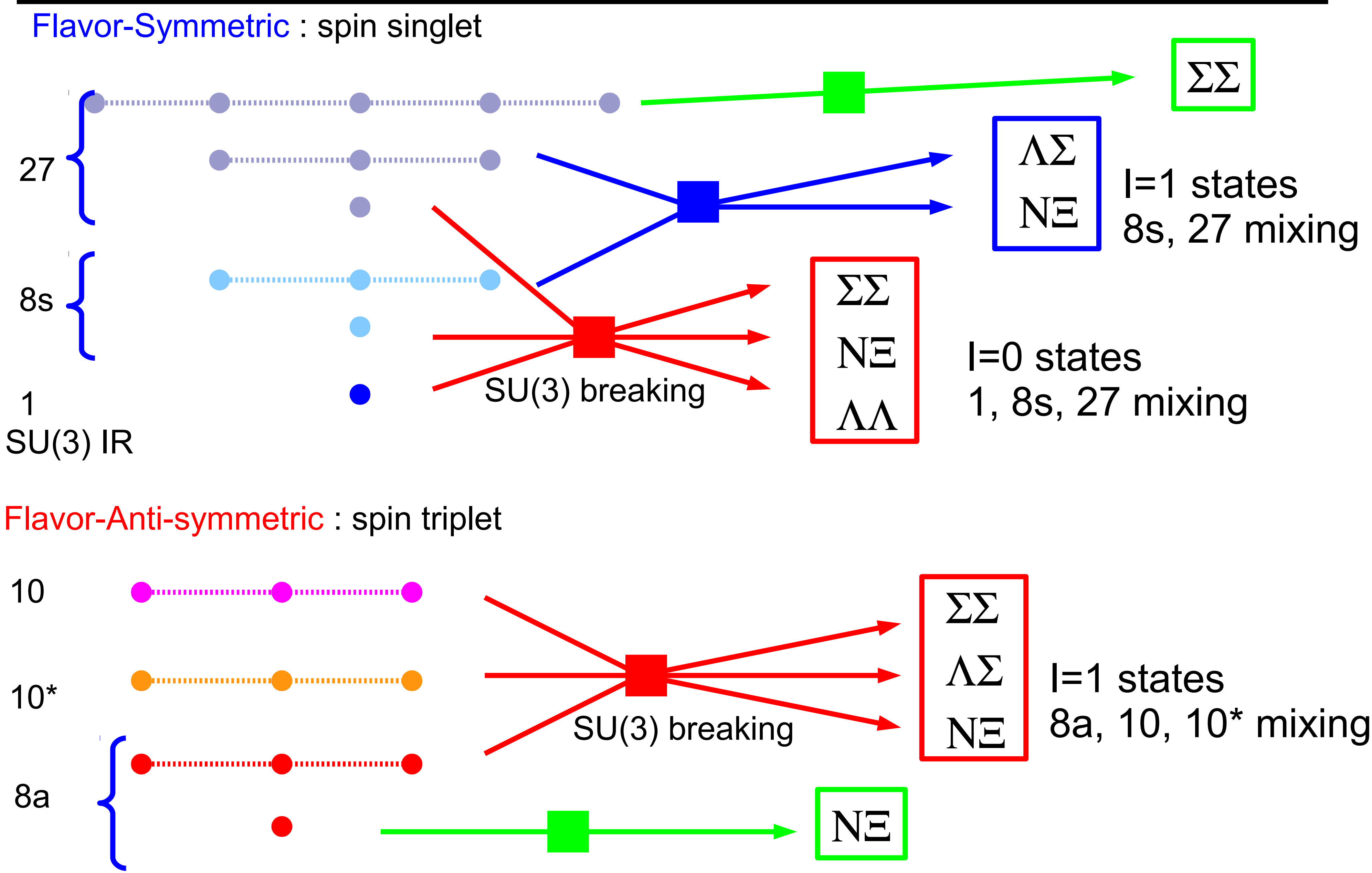
Flavor-Symmetric : spin singlet



Flavor-Anti-symmetric : spin triplet

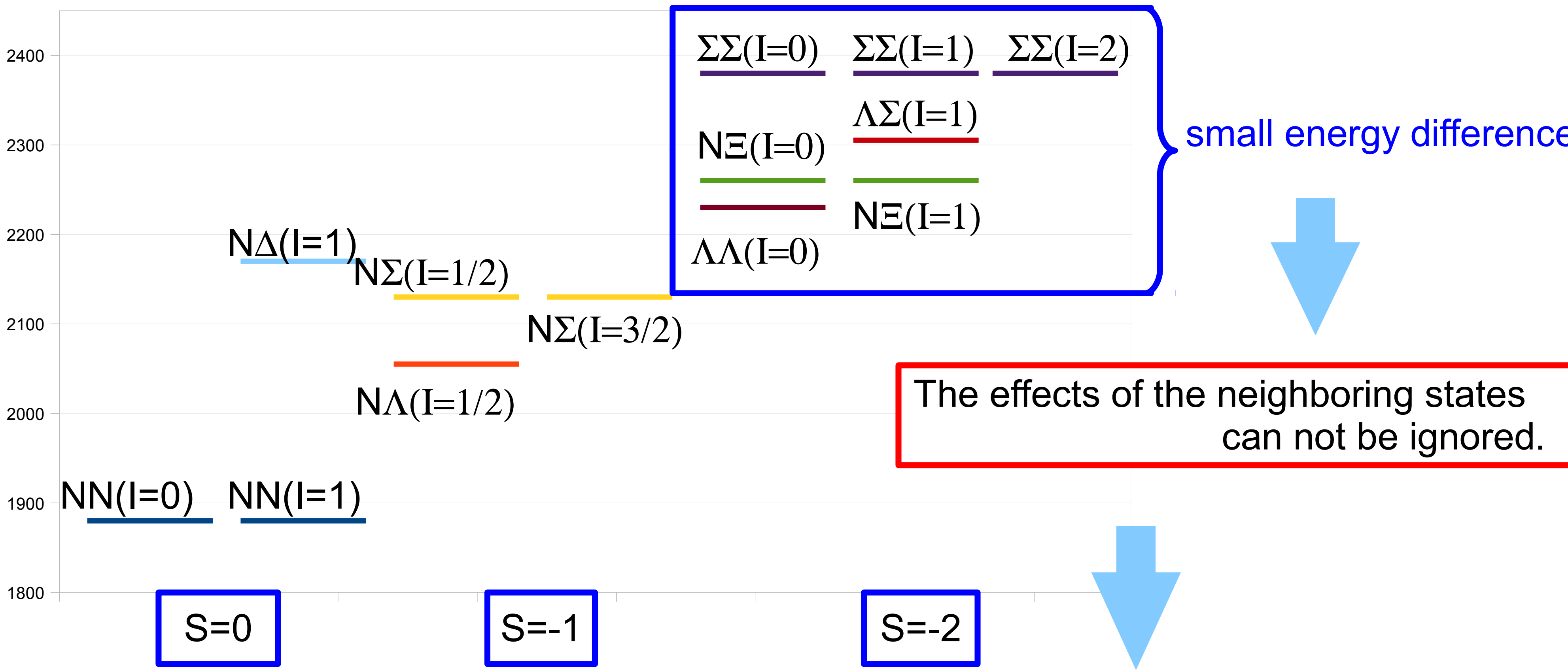


Classification of B-B states with S=-2



Channel coupling

Energy levels of baryon-baryon system in the real world



The effects of the neighboring states can not be ignored.

We have to extend our method to the coupled channel formalism.

HAL QCD strategy

- Calculate Bethe-Salpeter (BS) wave function on any gauge configuration.

$$\Psi(t-t_0,\vec{x}) = \sum_{\vec{y}} \langle 0|B(t,\vec{x}+\vec{y})B(t,\vec{x})|BB(t_0)\rangle$$

- Define the non-relativistic Schrödinger equation (general form)

$$\left(E-\frac{\nabla^2}{2\mu}\right)\Psi(\vec{x}) = \int U(\vec{x}-\vec{y})\Psi(\vec{y})d^3y$$

- Performing the derivative expansion for the interaction kernel

$$U(\vec{x}-\vec{y}) \simeq V_0(\vec{x})\delta(\vec{x}-\vec{y}) + V_1(\vec{x},\nabla)\delta(\vec{x}-\vec{y}) \cdots$$

- The potential is given as

$$V(\vec{x}) = E - \frac{1}{2\mu}\frac{\nabla^2\Psi(\vec{x})}{\Psi(\vec{x})}$$

- This technique is widely applicable for hadronic systems

Extention to the YN and YY systems

Coupled channel Schrödinger equation

Using four-point correlator *W* with an optimized source such as,

$$W_\alpha(\vec{x},E) = A\,\Psi_\alpha(\vec{x},E)\,e^{-Et}$$

The coupled channel Schrödinger equation can be rewritten as

$$\left(\frac{p_\alpha^2}{2\mu_\alpha} - H_0^\alpha\right) W_\alpha(\vec{x},E) = V_{\alpha\alpha}(\vec{x}) W_\alpha(\vec{x},E) + V_{\alpha\beta}(\vec{x}) W_\beta(\vec{x},E) + V_{\alpha\gamma}(\vec{x}) W_\gamma(\vec{x},E)$$

Define

$$R_\alpha(\vec{x},E) \equiv \frac{W_\alpha(\vec{x},E)}{C_\alpha(t)} \propto \exp\left(-(E-M_\alpha)t\right) \simeq \exp\left(-\frac{p_\alpha^2}{2\mu_\alpha}t\right)$$

Product of single baryon correlators

Taking time derivative of R,

$$\partial_t R_\alpha(\vec{x},E) = -\frac{p_\alpha^2}{2\mu_\alpha} R_\alpha(\vec{x},E)$$

Thus the potential matrix can be obtained as

$$\begin{pmatrix} V^{\Lambda\Lambda}_{\Lambda\Lambda}(\vec{x}) \\ V^{\Lambda\Lambda}_{N\Xi}(\vec{x}) \\ V^{\Lambda\Lambda}_{\Sigma\Sigma}(\vec{x}) \end{pmatrix} = \begin{pmatrix} W_{\Lambda\Lambda}(\vec{x},E_0) & W_{N\Xi}(\vec{x},E_0) & W_{\Sigma\Sigma}(\vec{x},E_0) \\ W_{\Lambda\Lambda}(\vec{x},E_1) & W_{N\Xi}(\vec{x},E_1) & W_{\Sigma\Sigma}(\vec{x},E_1) \\ W_{\Lambda\Lambda}(\vec{x},E_2) & W_{N\Xi}(\vec{x},E_2) & W_{\Sigma\Sigma}(\vec{x},E_2) \end{pmatrix}^{-1} \begin{pmatrix} -C_{\Lambda\Lambda}\partial R_{\Lambda\Lambda}(\vec{x},E_0) - H_0^\alpha W_{\Lambda\Lambda}(\vec{x},E_0) \\ -C_{\Lambda\Lambda}\partial R_{\Lambda\Lambda}(\vec{x},E_1) - H_0^\alpha W_{\Lambda\Lambda}(\vec{x},E_1) \\ -C_{\Lambda\Lambda}\partial R_{\Lambda\Lambda}(\vec{x},E_2) - H_0^\alpha W_{\Lambda\Lambda}(\vec{x},E_2) \end{pmatrix}$$

Numerical setup

- 2+1 flavor gauge configurations by CP-PACS/JLQCD collaboration.
  - RG improved gauge action & *O(a)* improved clover quark action
  - β = 1.83, a^{-1} = 1.632 [*GeV*], a = 0.1209 [*fm*]
  - 16^3x32 lattice, L = 1.934 [*fm*].
  - κ_{ud} = 0.13825, κ_s = 0.13710 was chosen (named Set3).
  - 800 / 800 configurations are used.
- Flat wall source is considered to produce S-wave B-B state.
  - 16 shifted sources every 2 time-slices are considered to enhance the S/N ratio.
- The USQCD computer resources are used.
  - We acknowledge the USQCD for providing of computer resources.

	π	K	m_π/m_K	N	Λ	Σ	Ξ
Set3	661±1	768±1	0.860	1482±3	1557±3	1576±3	1640±3

In unit of MeV

Isospin combinations of BB operator

$\Lambda\Lambda,\ p\Xi^-,\ n\Xi^0,\ \Xi^- p,\ \Xi^0 n,\ \Sigma^+\Sigma^-,\ \Sigma^0\Sigma^0,\ \Sigma^-\Sigma^+,\ \Lambda\Sigma^0,\ \Sigma^0\Lambda$

I=0 operators

$$\Lambda\Lambda$$

$$N\Xi = +\sqrt{\frac{1}{2}}\, p\Xi^- - \sqrt{\frac{1}{2}}\, n\Xi^0$$

$$\Sigma\Sigma = +\sqrt{\frac{1}{3}}\,\Sigma^+\Sigma^- - \sqrt{\frac{1}{3}}\,\Sigma^0\Sigma^0 + \sqrt{\frac{1}{3}}\,\Sigma^-\Sigma^+$$

Flavor symmetric

I=1 operators

$$N\Xi = +\sqrt{\frac{1}{2}}\, p\Xi^- + \sqrt{\frac{1}{2}}\, n\Xi^0$$

$$\Sigma\Sigma = +\sqrt{\frac{1}{2}}\,\Sigma^+\Sigma^- - \sqrt{\frac{1}{2}}\,\Sigma^-\Sigma^+$$

$$\Lambda\Sigma$$

Flavor anti-symmetric

I=2 operators

$$\Sigma\Sigma = +\sqrt{\frac{1}{6}}\,\Sigma^+\Sigma^- + \sqrt{\frac{4}{6}}\,\Sigma^0\Sigma^0 + \sqrt{\frac{1}{6}}\,\Sigma^-\Sigma^+$$

Lists of channels

I=0 states

Spin	BB channels			SU(3) representation		
$^1S_0$	$\Lambda\Lambda$	$N\Xi$	$\Sigma\Sigma$	1	8s	27
$^3S_1$	--	$N\Xi$	--	8a	--	--

Strong attraction (H-dibaryon)

I=1 states

Attraction

Strong repulsion

Spin	BB channels			SU(3) representation		
$^1S_0$	$N\Xi$	--	$\Lambda\Sigma$	--	8s	27
$^3S_1$	$N\Xi$	$\Sigma\Sigma$	$\Lambda\Sigma$	8a	10	10*

Similar to The NN potential

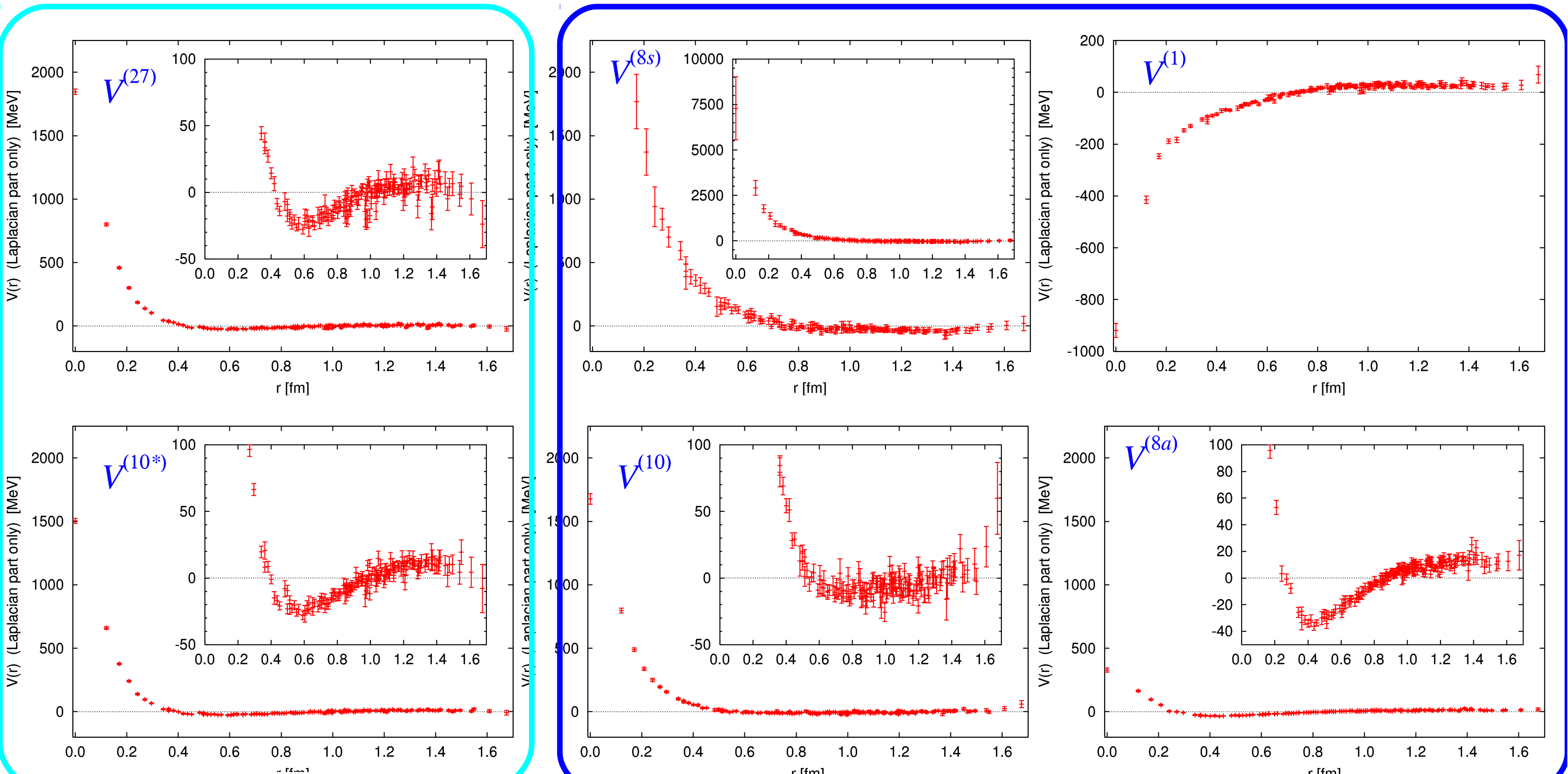
I=2 states

Repulsion

Spin	BB channels			SU(3) representation		
$^1S_0$	$\Sigma\Sigma$			--	--	27
~~$^3S_1$~~						

T. Inoue

Baryon-baryon potential in the flavor SU(3) limit.

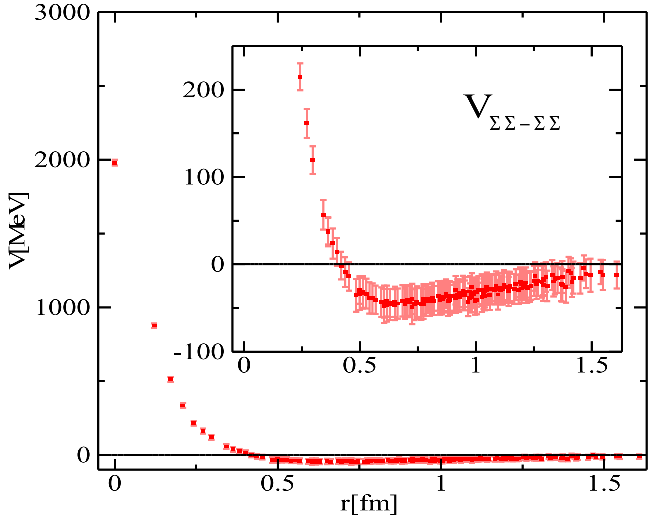


Accessible by non-strange sector

Strong flavor dependence turns out with irreducible rep.
Various interaction are seen by extending to SU(3).

ΣΣ (I=2) $^1S_0$ channel

Set 3 : m_π= 661

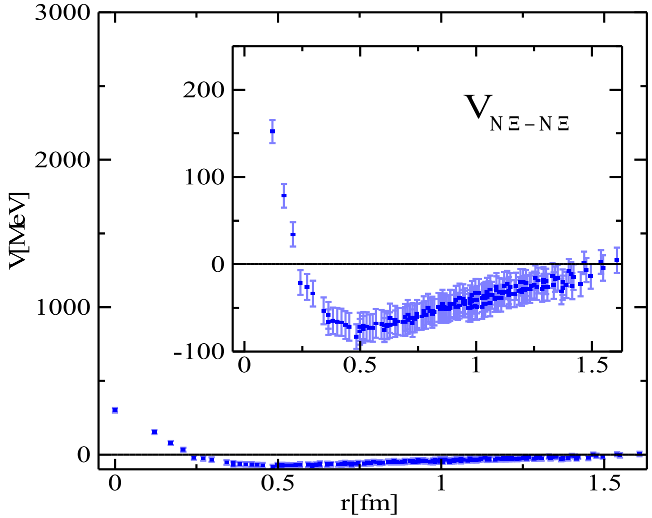


Direct correspondence to the 27plet
in SU(3) irreducible representation

Similar behavior to the NN potential

Short range repulsion and mid-range attraction

NΞ (I=0) $^3S_1$ channel



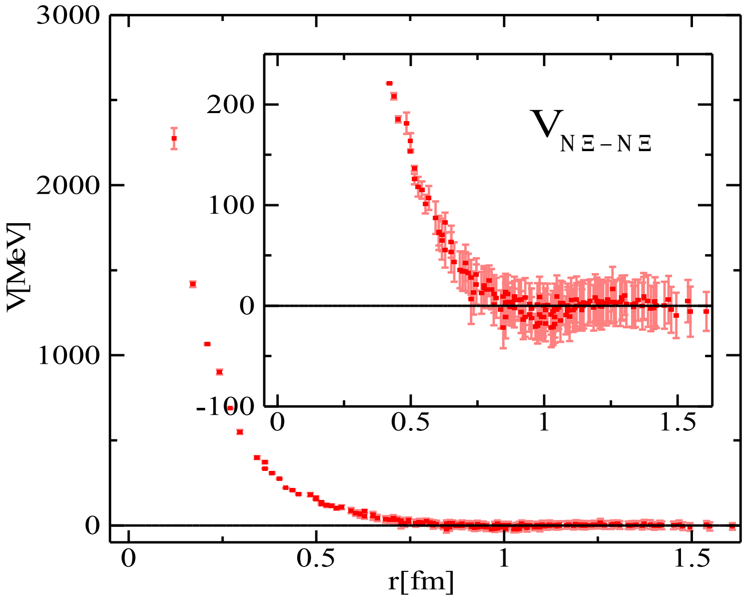
Direct correspondence to 8_aplet.

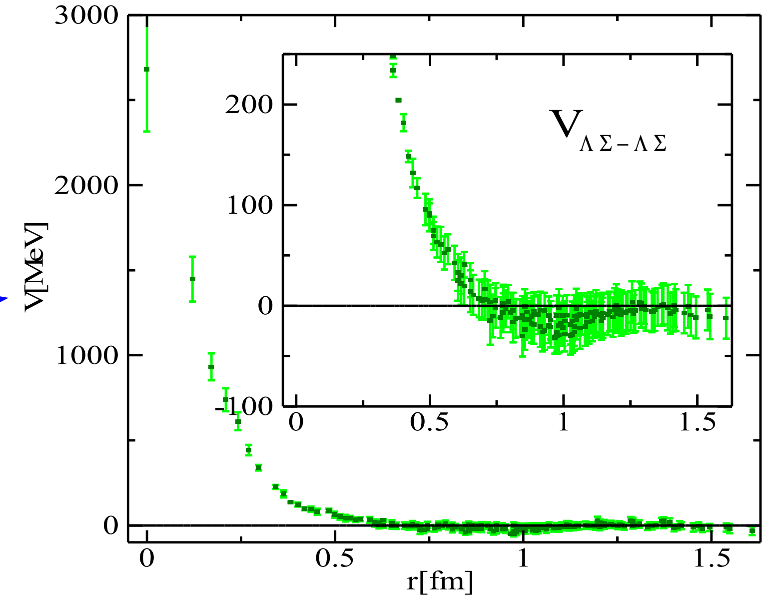
Repulsive core is not so high

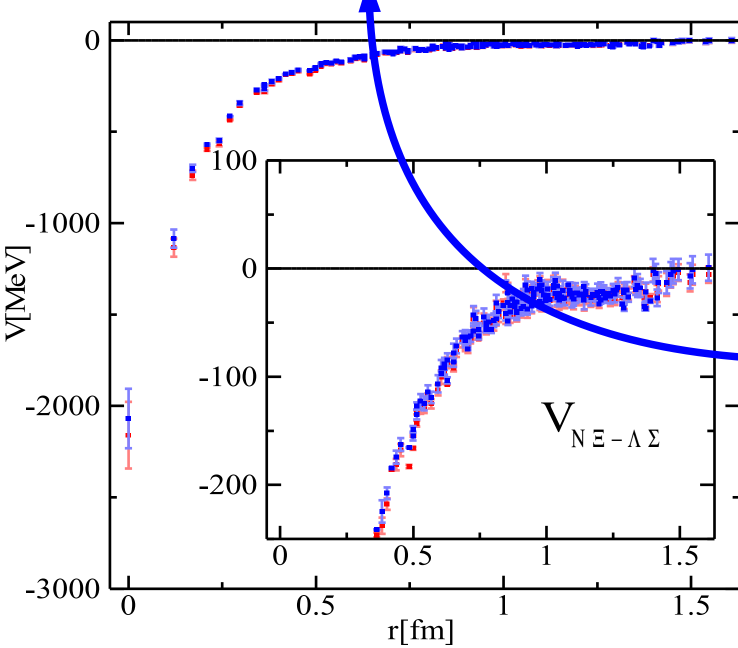
More attractive than 27 plet potential

$N\Xi$, $\Lambda\Sigma$ (I=1) $^1S_0$ channel

Set 3 : m_π= 661







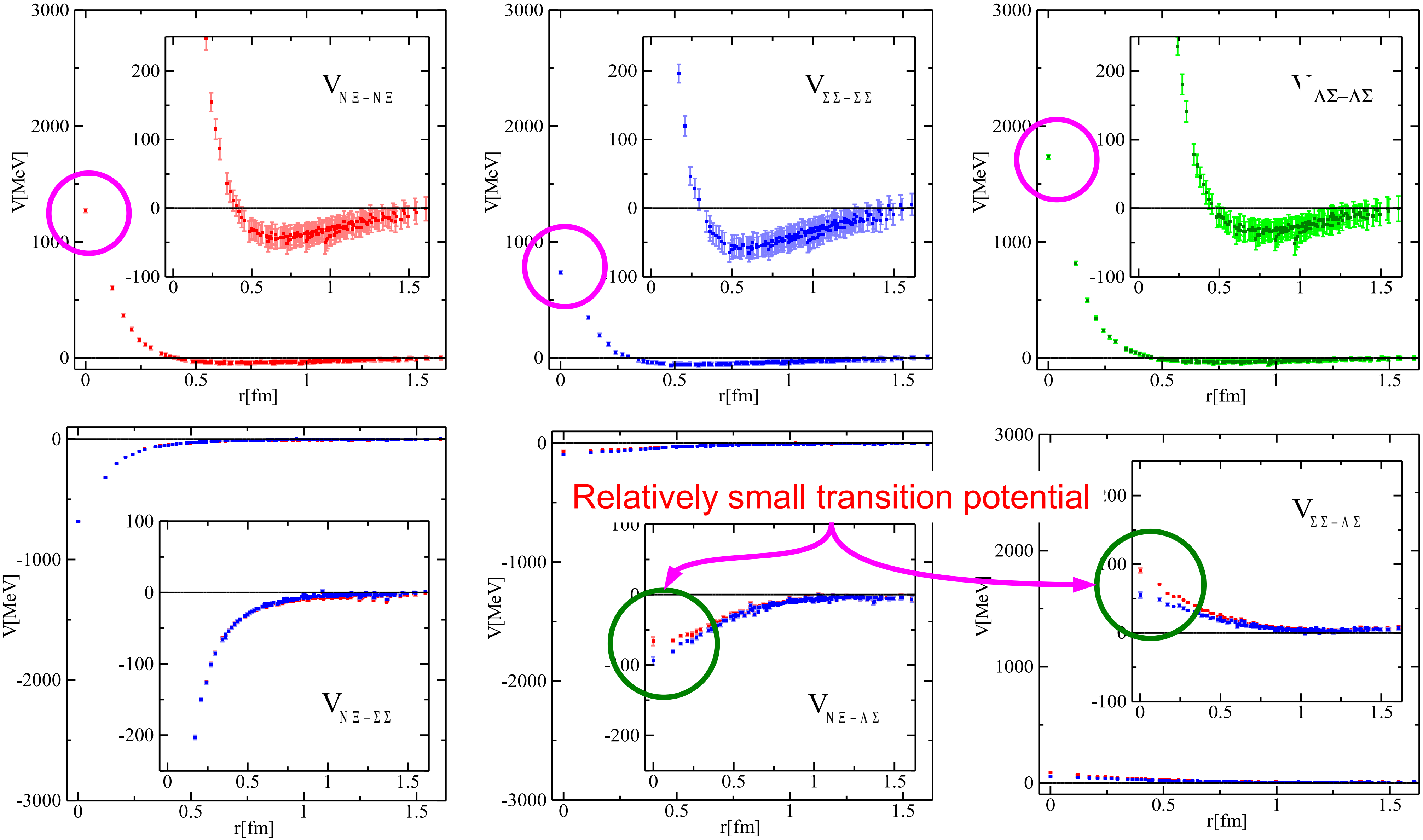
8s-27 mixing channel.

Origin of strong repulsion

No attractive pocket in diagonal elements

$N\Xi$, $\Sigma\Sigma$, $\Lambda\Sigma$ ($I=1$) $^3S_1$ channel

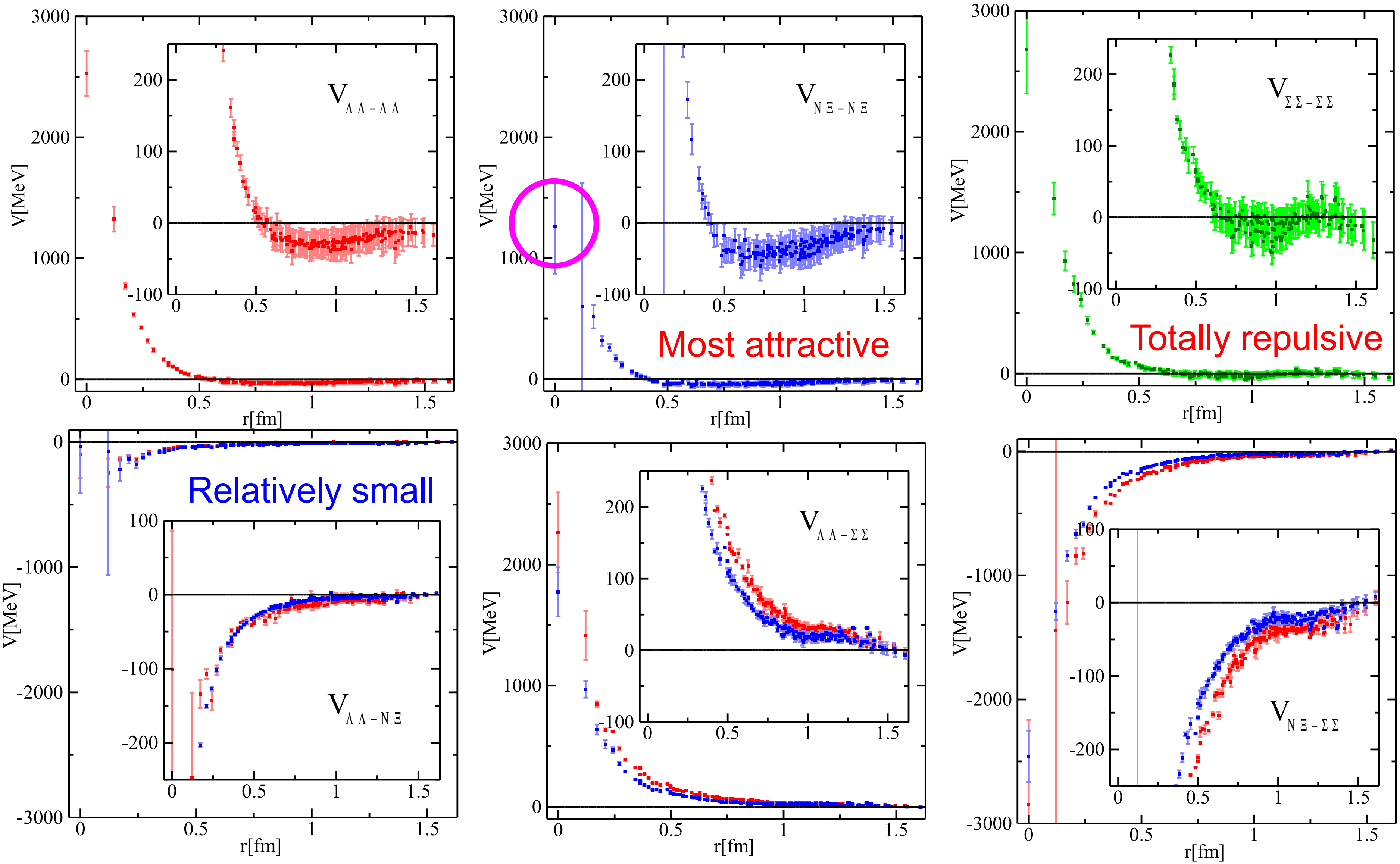
Set 3 : m_π= 661



Coupling of $\Lambda\Sigma$ state to the other states are quite small.

ΛΛ, NΞ, ΣΣ (I=0) $^1S_0$ channel

Set 3 : m_π= 661



In this channel, our group found the "H-dibaryon" in the SU(3) limit.

T. Inoue [HAL QCD coll.]PRL106(2011)162002.

Lists of channels

I=0 operators

Spin dependence of potential

Spin	BB channels			SU(3) representation		
$^1S_0$	$\Lambda\Lambda$	$N\Xi$	$\Sigma\Sigma$	1	8s	27
$^3S_1$	--	$N\Xi$	--	8a	--	--

I=1 operators

Isospin dependence of potential

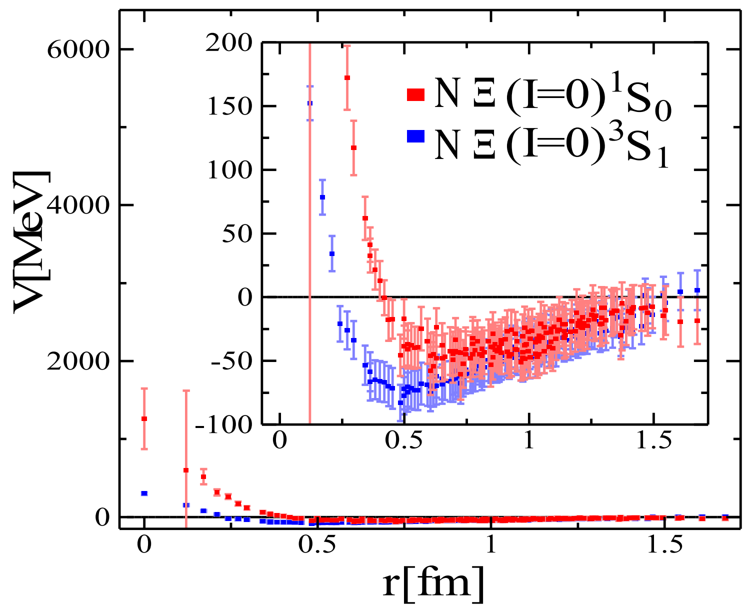
Spin	BB channels			SU(3) representation		
$^1S_0$	$N\Xi$	--	$\Lambda\Sigma$	--	8s	27
$^3S_1$	$N\Xi$	$\Sigma\Sigma$	$\Lambda\Sigma$	8a	10	10*

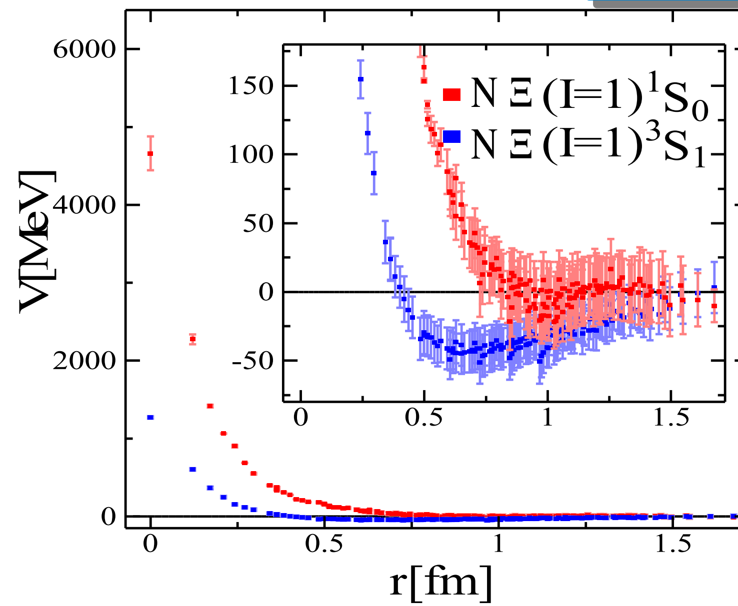
I=2 operators

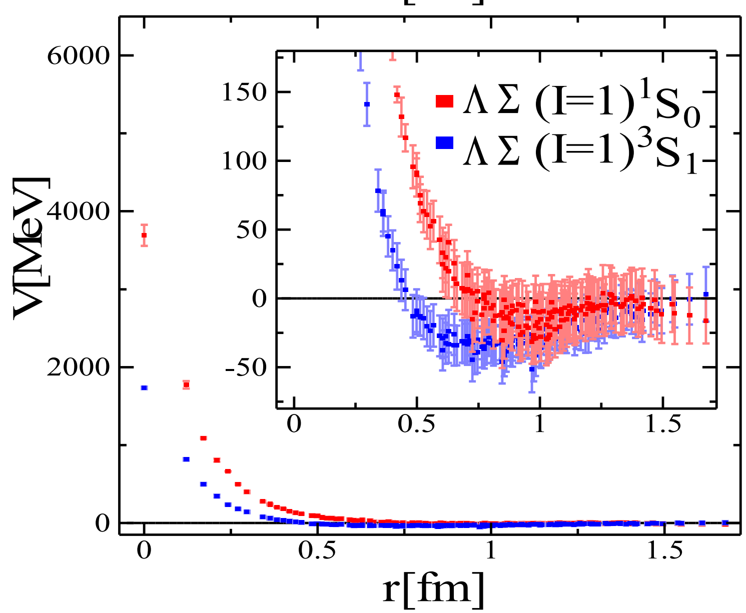
Spin	BB channels			SU(3) representation		
$^1S_0$	$\Sigma\Sigma$			--	--	27
$^3S_1$						

Spin dependence of NΞ, ΛΣ potentials

Set 3 : m_π= 661





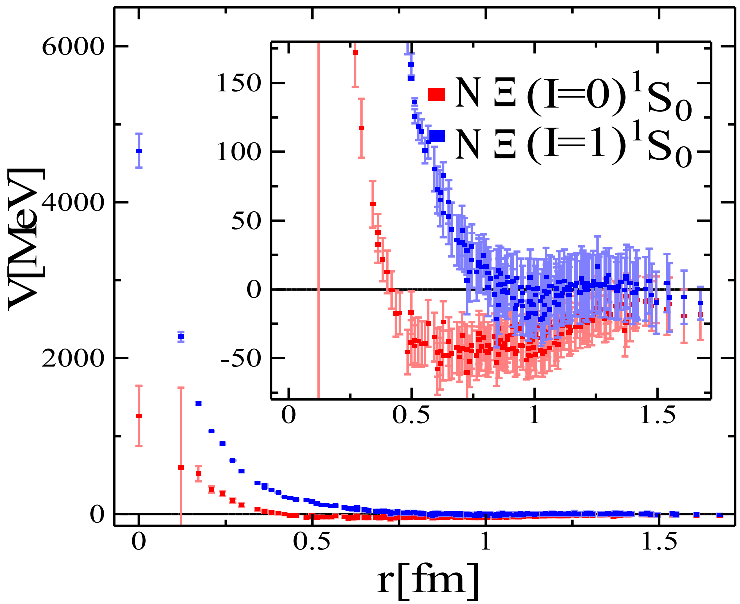


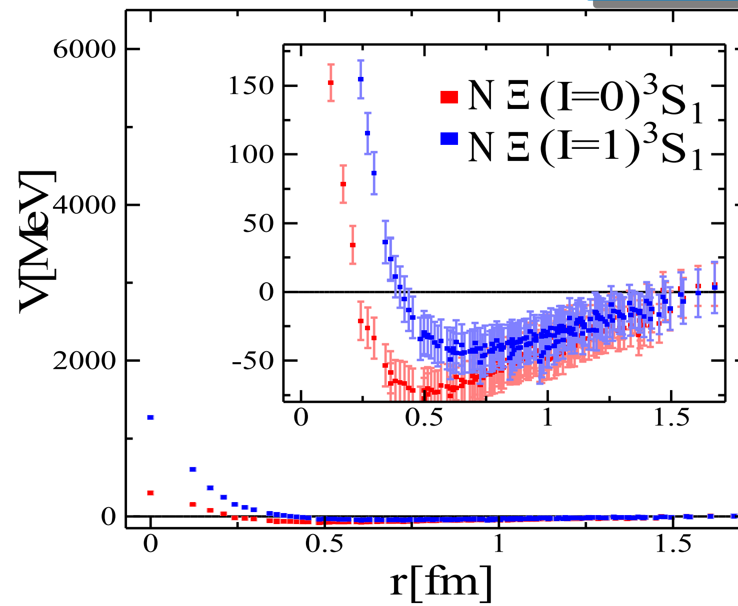
Spin triplet potentials are more attractive than the spin singlet potentials

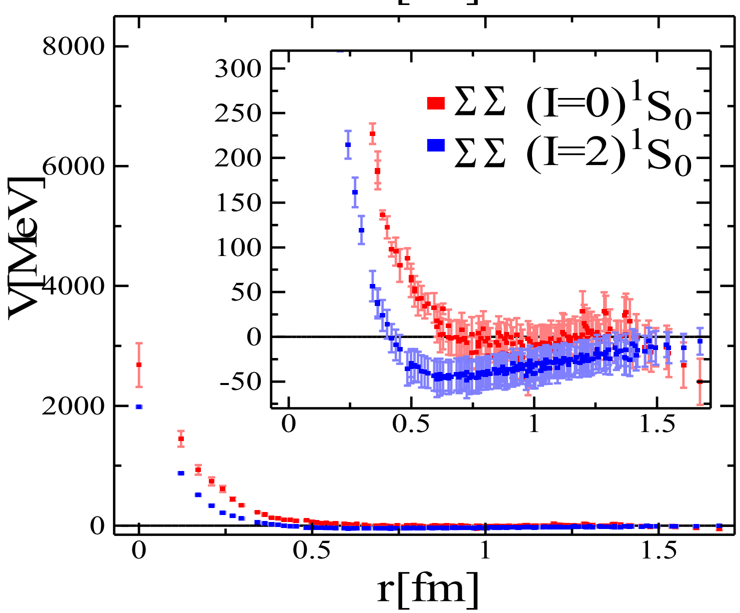
The tensor potential is not separated yet In spin triplet channel.

Isospin dependence of NΞ, ΣΣ potentials

Set 3 : mπ= 661







In NΞ potentials the I=0 potentials are more attractive than the I=1 potentials.

The short range behavior of potentials are strongly depend on the choice of state.

Summaries and outlooks

- We have investigated the S=-2 BB interactions from lattice QCD.
  - They are complement to experiments.
- In order to deal with a variety of interactions, we extend our method to the coupled channel formalism.
  - Asymptotic momentum can be determined from time derivative of R-correlator.
  - The source optimization is not necessary in our formalism by employing the time derivative treatment of the energy part.
- We have found the strong state dependence of NΞ potential especially at the short range region.

- Realistic potentials with lighter quark masses and large volume
  - Toward the physical point !
  - Look for the physical “H-dibaryon”.
- Coupled channel technique is powerful and widely applicable
  - We will tackle to reveal all baryon-baryon interactions with S=0, -1, … -6 below the pion production threshold.

By the world's fastest “K computer”