A New Equation of State with Abundances of All Nuclei For Core-Collapse Simulations

(重力崩壊コア内の原子核組成を含む状態方程式)

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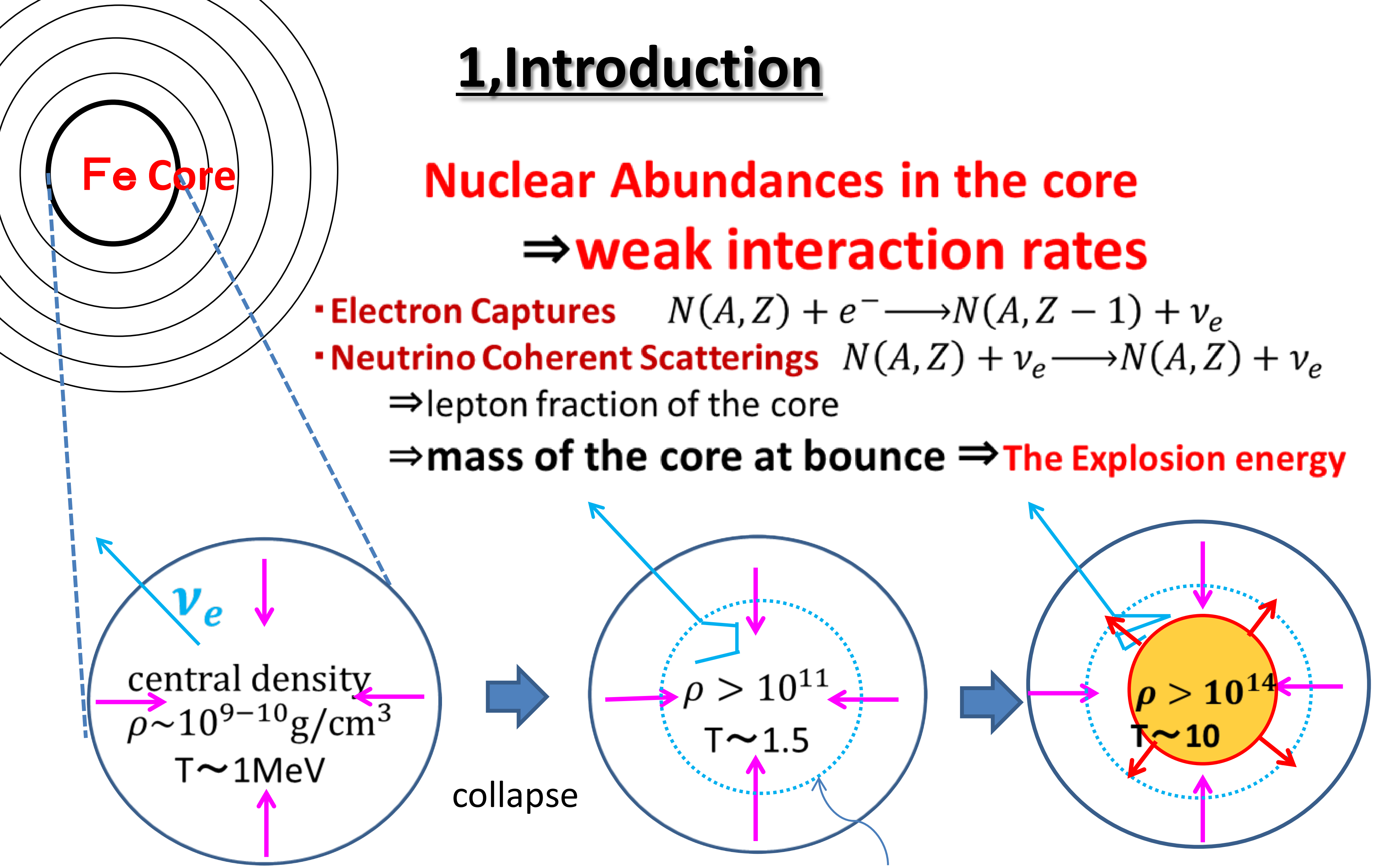
1,Introduction



Nuclear Abundances in the core

⇒weak interaction rates

- **Electron Captures** $N(A,Z)+e^-\longrightarrow N(A,Z-1)+\nu_e$
- **Neutrino Coherent Scatterings** $N(A,Z)+\nu_e\longrightarrow N(A,Z)+\nu_e$

⇒lepton fraction of the core

⇒mass of the core at bounce ⇒The Explosion energy

Neutrino Sphere

The core of massive stars (M>10Msolar)

Bounce ($\rho=\rho_0$)
⇒Shock Wave

Supernovae

Previous works and Motivation

●Standard EOSs

1,Lattimer et al.(1991)

- SKyerm Type Interaction & Compressible Liquid Drop model

2,Shen et al.(1998)

- Relativistic Mean Field(RMF) & Thomas Fermi approximation

Only one heavy nucleus (Single Nucleus Approximation(SNA))

For actual calculations of weak interaction rates,

⇒ **we need the EOS with multi species of nuclei.（Motivation）**

●Multi-Nucleus EOS

3,Hempel et al. (2009)

- Nuclear Statistical Equilibrium(NSE) & RMF

- Nuclear proton numbers $Z < 100$

(EOSs of SNA predict more heavy nuclei is formed at high densities.)

- Only Coulomb correction on nuclear masses in Vacuum.

(Bulk & Surface energies are adapted from values in vacuum even at high ρ & T.)

2, EOS model

Baryonic EOS with Multi Species of Nuclei (Z<1000)

EOS table

Density ρ	$:10^{5}\sim10^{16}\ \mathrm{g/cm^{3}}$	220 Mesh in log10
Temperature T	$:10^{-1}\sim10^{2.2}$ MeV	128 Mesh in log10
Proton fraction Yp	: 0.01 ～0.65	130 Mesh

･Formulations

NSE (Nuclear Statistical equilibrium)

$T \ >\sim 5\times10^{9}$ [K]

[time scale of all nuclear interactions]
< [dynamical time scale]

$$(A,Z)\leftrightarrow(A-1,Z)+n$$
$$(A,Z)\leftrightarrow(A-1,Z-1)+p$$

We solved the minimized Free Energy *F* (*X*p, *X*n, {*X*i})
for Abundances *X*p, *X*n, {*X*i} on given *ρ*, *T* & *Y*p .
⇒ Thermodynamics Quantities & Abundances *X*p, *X*n, {*X*i}

The Free Energy density

$$f = \underset{\text{free nucleons}}{f_{p,n}} + \underset{\text{nuclei}}{\sum_i} n_i\{\underset{\text{Translational Energy}}{E_i^{trans}} + \underbrace{E_i^{bulk} + E_i^{surf} + E_i^{coul}}_{\text{Nuclear Mass}}\}$$

Free Nucleons: **RMF theory (TM1 parameter set)**

Nuclei: A Mass Formula based on **Liquid Drop Model**
(including **Nuclear Shell term**
& approximate **nuclear pasta phase**)

The points of the free energy

@Low densities : **Boltzmann gasses**
with experimental mass data (ordinary NSE)

@ Saturation density : **a continuous transition**
to the EOS for supra-nuclear density (**RMF**)

$$f = \underline{f_{p,n}} + \sum_i n_i \{ \underline{E_i^{trans}} + \underline{E_i^{bulk} + E_i^{surf} + E_i^{coul}} \}$$

Free nucleons
⇒**RMF** theory
+ excluded volume effect

Translational
⇒**Boltzmann gas**
+ approximate exclude volume

Nuclear Mass
⇒**Modified Liquid Drop Model**

Modified Liquid Drop Model for Nuclei (1 < Z < 1000)

- **Bulk Energy**: **RMF** and **Mass data** (at low densities)

$$\boxed{E_i^B = M_i^{\mathrm{data}} - [E_i^C]_{\mathrm{vacuum}} - [E_i^S]_{\mathrm{vacuum}}}$$

- **Surface**: Surface tensions σ_i (Lattimer 1991) +high density correction

$$\boxed{E_i^s = \sigma_i \times (surface\ area) \times (1 - n_{nucleon}/n_{si})^2}$$

n_{si}: nuclear saturation density

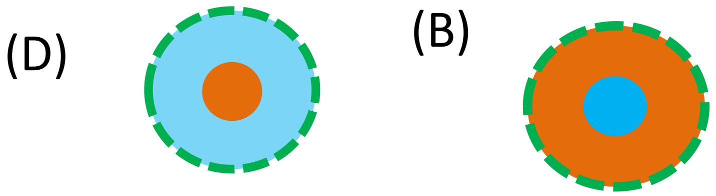
- **Coulomb** :Integration of Coulomb force in WS cell

The cell volume V_i of each nucleus ⇐ **the charge neutrality** $(e^-, p \& Z_i)$

$$\boxed{u_i = V_i^N / V_i} \qquad \boxed{V_i^N = \frac{A_i}{n_{si}}}$$

V_i:Cell V_i^N:nucleus **Free nucleons**

u<0.3 :Droplet(D)
0.3<u<0.7 : other Pasta (interpolation)
u>0.7 :Bubble(B)



The Limits of Free Energy Density

@Low densities

nuclear mass

$$f = \underline{f_{p,n}} + \sum_i n_i\{\underline{E_i^{trans}} + \underline{\boxed{E_i^{bulk} + E_i^{surf} + E_i^{coul}\}}}$$

⇒Boltzmann **⇒Boltzmann** **⇒～Mass data**

@nuclear saturation density

$$f = \underline{f_{p,n}} + \sum_i n_i\{\underline{E_i^{trans}} + \boxed{\underline{E_i^{bulk}} + \underline{E_i^{surf} + E_i^{coul}}\}}$$

⇒RMF **⇒0 (exclude volume)** **⇒RMF** **⇒0 (bubble volumes ⇒ 0)**

The Comparison with other EOS

$$f = f_{p,n} + \cancel{\sum_{\bigcirc\!\!\!\!i}} n_i\{\cancel{E_i^{trans}} + \underline{E_i^{bulk} + E_i^{surf}} + E_i^{coul}\}$$

Shen EOS : Single nucleus

× nuclear translation

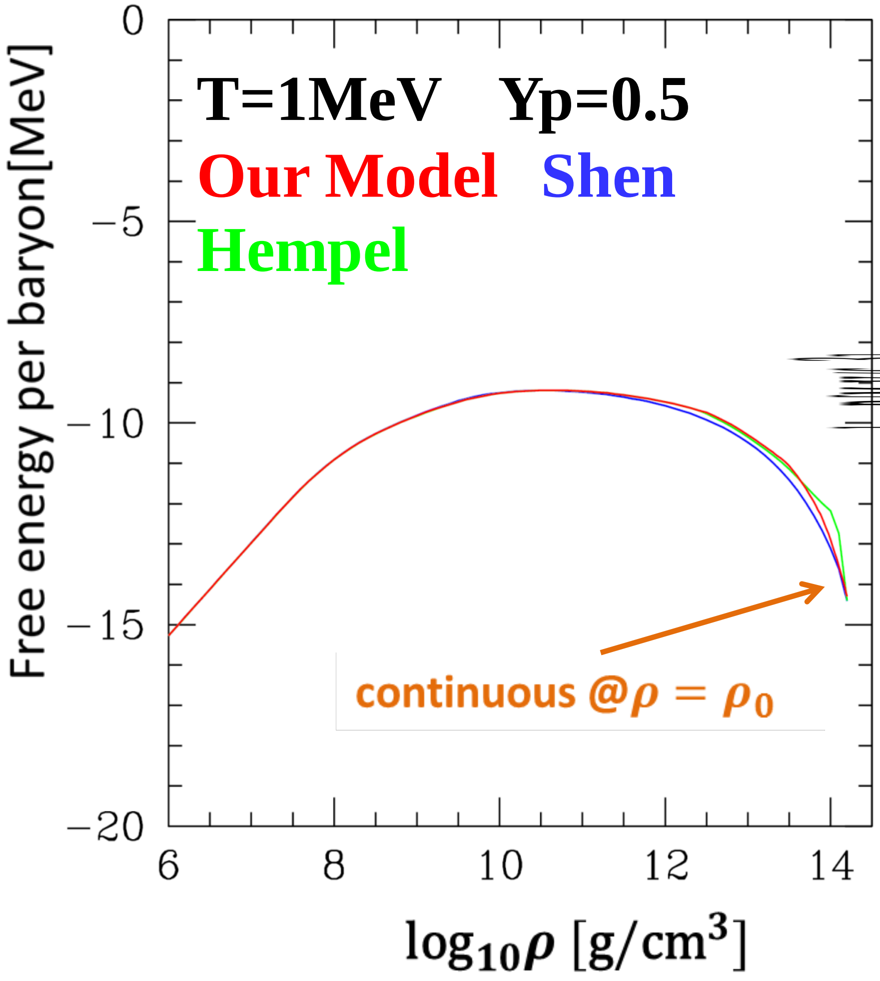
Hempel: only z<100 (our EOS Z<1000)

△Nuclear Mass :only coulomb corrections

3,Results

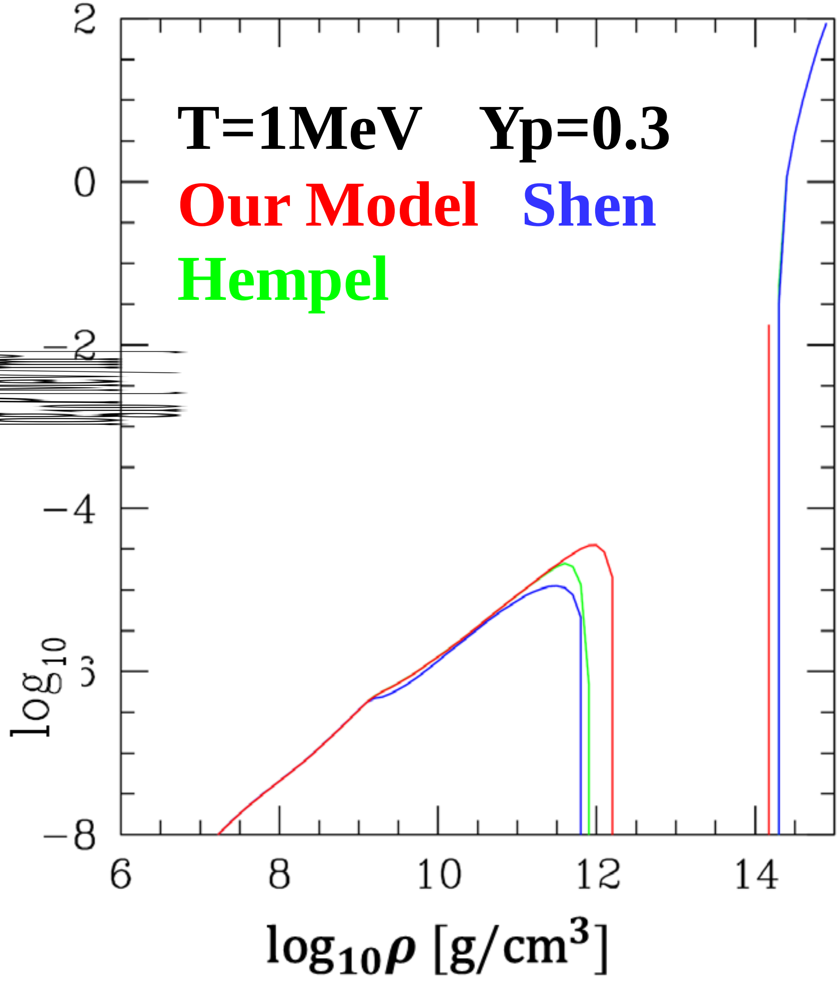
Thermodynamical quantities are not very different from other EOSs.

①Free Energy/baryon



continuous $@\rho = \rho_0$

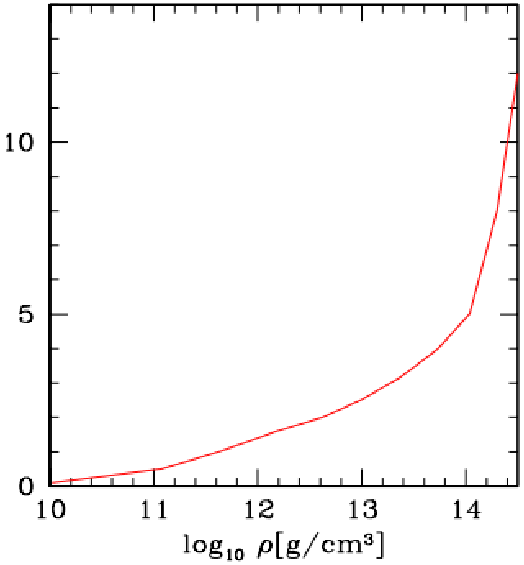
②baryon pressure



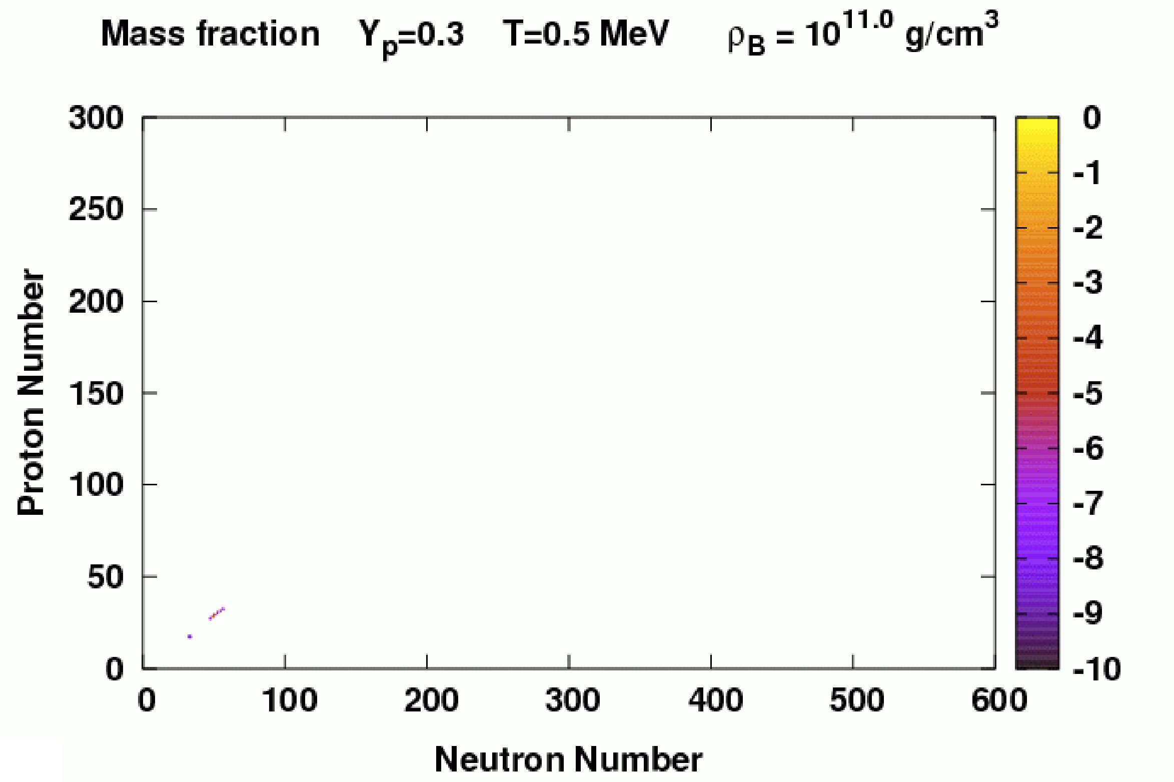
③ Mass fractions of nuclei in the (N, Z) plane along with adiabatic line ($S_{baryon} = 1\ [k_{\mathrm{B}}]$)

Adiabatic line
Yp=0.3
($S_{baryon} = 1\ [k_{\mathrm{B}}]$)

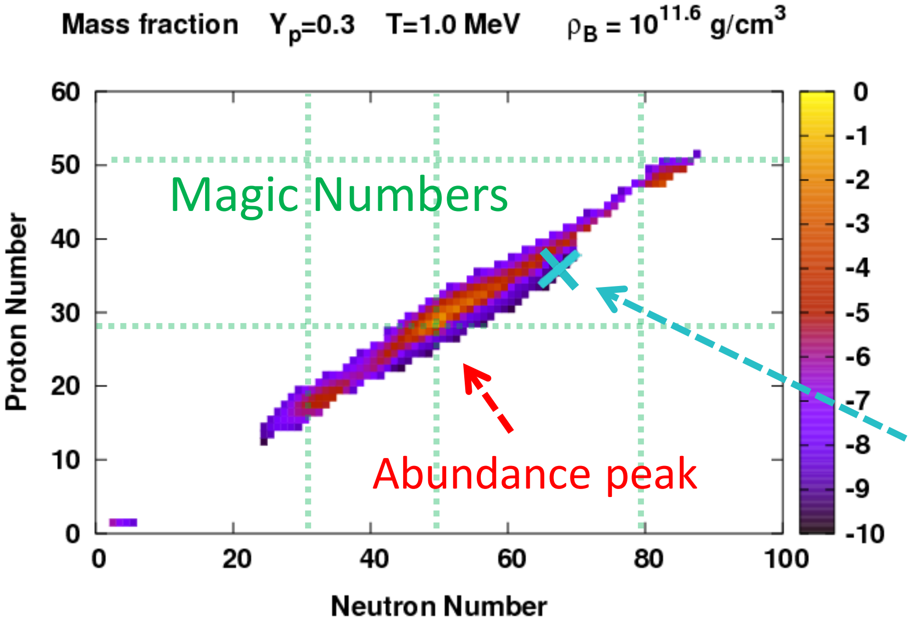
Temperature[MeV]



$\log_{10}\rho\ [\mathrm{g/cm^3}]$

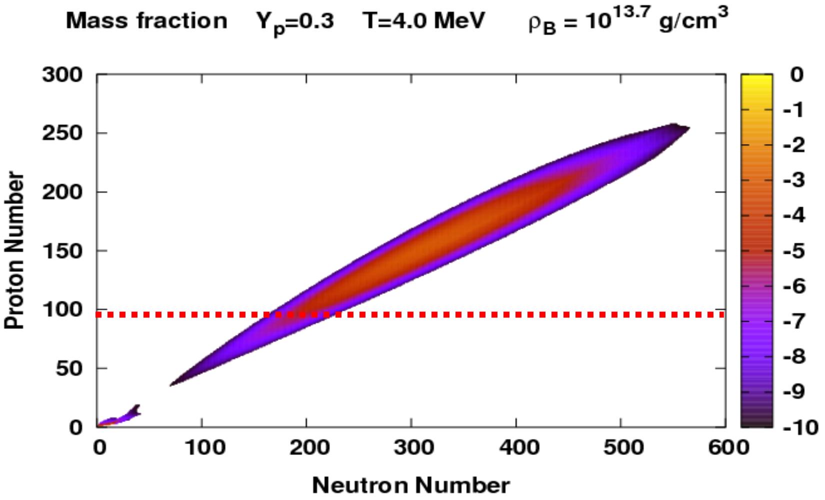


Mass fractions of nuclei （log10） in the (N, Z) plane



Nuclei are abundant
in Vicinities of Magic Numbers.
Abundance peak is
different from Shen EOS .

Representative nucleus
of Shen’s EOS (SNA)



Heavier nuclei (Z>100) than
upper limit of Hempel EOS

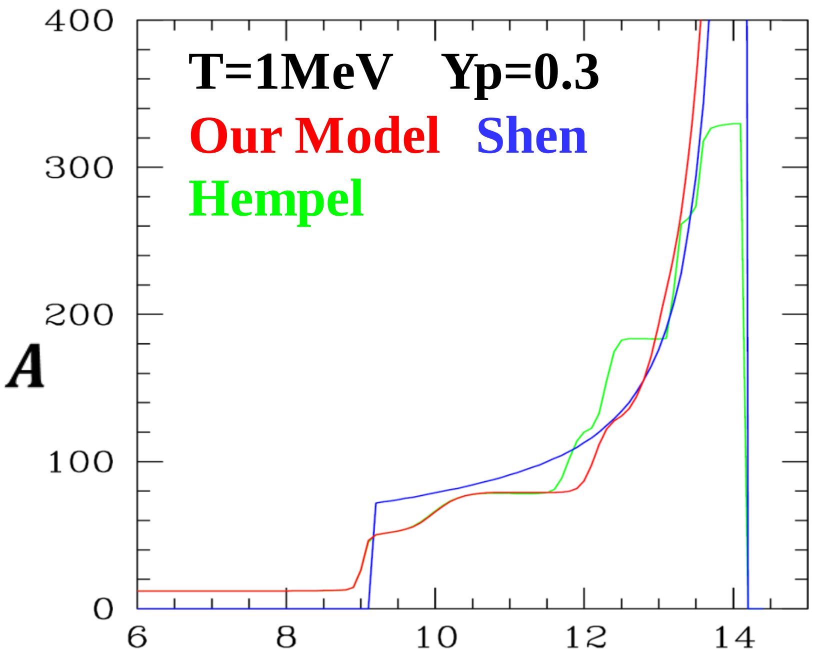
③Nuclear average mass number

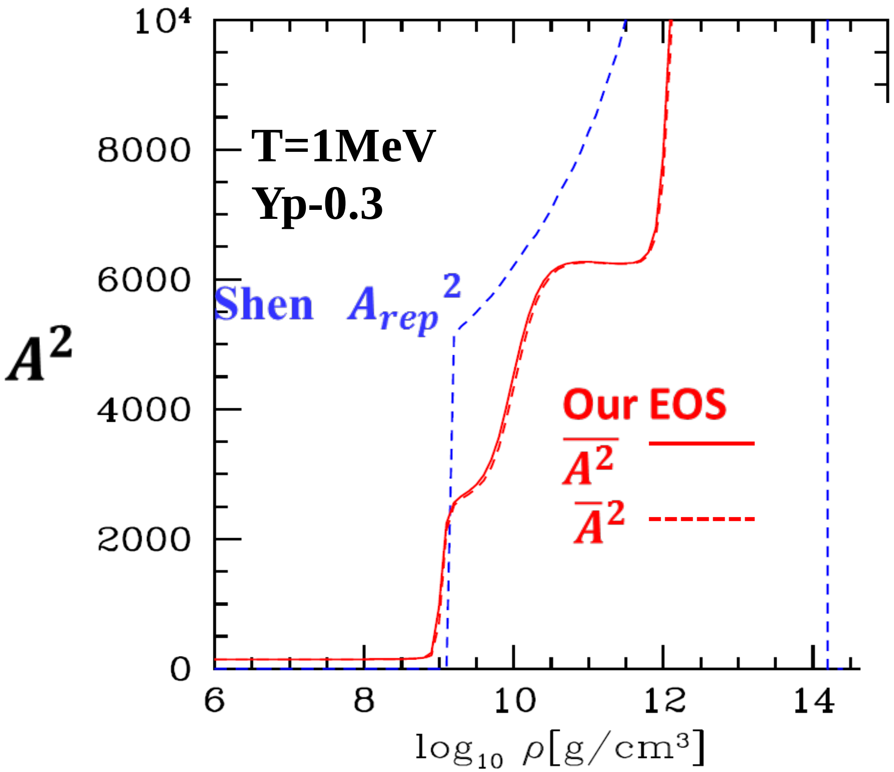
$N(A,Z)+\nu \longrightarrow N(A,Z)+\nu$ **depend on A^2**

We need average of squared mass number $\overline{A^2}$

Shen EOS (SNA) ⇒ square of average mass number $A_{rep}{}^2$

We investigated $\overline{A^2} \neq \overline{A}^2$





- **Shell Term (our model,Hemple)**
- **Nuclei can't grow @$\rho\sim 10^{13}$ (Hempel)**

$\overline{A^2}\sim\overline{A}^2$ (Dispersion is small)

⇒ Calculation from $\overline{A}^2$ is no problem

4, Summary

Model: NSE

Free Nucleons: RMF

Nuclei: a mass formula based on Liquid Drop Model
(including Nuclear Pasta Phase, Shell Term)

Result

- **Thermodynamical quantities are not very different from the Shen EOS.**
- **Average mass number is different. (∵ Shell term)**
  ⇒ It affects neutrino interactions in collapsing cores.

Next Steps

- **Binding Energies of Light Nuclei (deuterons, triton, Helium 3)**
- **Theoretical mass data for shell energy of heavier nuclei (KTUY 05)**
- **We replace the RMF theory by other uniform matter EOS (Takano & Togashi)**

	Lattimer	**H.Shen**	**Hempel**	**Our model**
Model	Skyrme+ Compressible LDM	RMF + Thomas Fermi	RMF + NSE+ mass data(+RMF)	RMF+ NSE+ +LDM +mass data
Component heavy nuc.	Single	Single	multi (Z<100)	multi (Z<1000)
Shell term	×	×	○	○
Nuclear Shape	Droplet +bubble	Droplet only	Droplet only	Droplet +bubble (+other)
$E_{surface}$ Correction	○	○	×	○

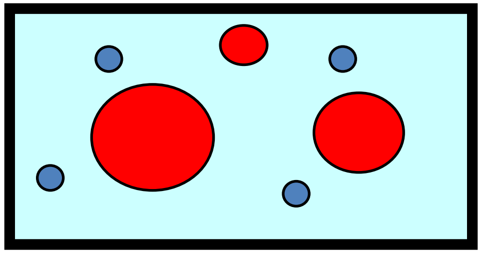
①自由核子

$$f = f_{p,n} + \sum_i n_i\{E_i^{trans} + E_i^{bulk} + E_i^{surf} + E_i^{coul}\}$$

- 核子は原子核による排除体積効果

  V:全体積　**V' :核子● の動き回れる体積(● 以外)**

- V'内を**RMF計算**



$$n'_{p/n} = \frac{N_{p/n}}{V'} \qquad \boxed{f'_{p,n} = f^{RMF}(n'_p, n'_n, T)} \qquad \eta = \frac{V'}{V} \qquad \boxed{f_{p,n} = \eta f'_{p,n}}$$

V'内のFree Energy Density

V内のFree Energy Density

②原子核の並進運動項

$$f = f_{p,n} + \sum_i n_i\{E_i^{trans} + E_i^{bulk} + E_i^{surf} + E_i^{coul}\}$$

- ボルツマン気体を仮定
- 内部自由度(スピン、励起状態) g_i ：温度Tの関数(Fai&Rundurp(1982)
- 原子核同士の排除体積による運動の制限 を 近似的に与える (Lattimer 1991)

$$\boxed{E^i_{\rm kine} = kT\underline{\left\{\log\left(\frac{n_i}{g_i(T)n_i^Q}\right) - 1\right\}}\,\underline{\left(1 - \frac{n_b}{n_s}\right)}}$$

Boltzmann gas

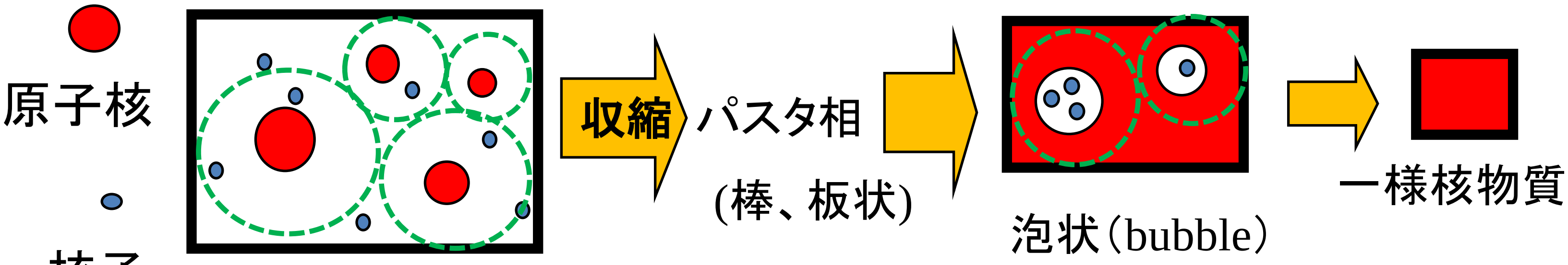
排除体積

原子核の質量

Wigner-Seiz Cell

$$f = f_{p,n} + \sum_i n_i\{E_i^{trans} + E_i^{bulk} + E_i^{surf} + E_i^{coul}\}$$

核原子核に荷電中立なWigner-Seiz Cellをとる。（全体積を原子核に割り振る）



球状（droplet）

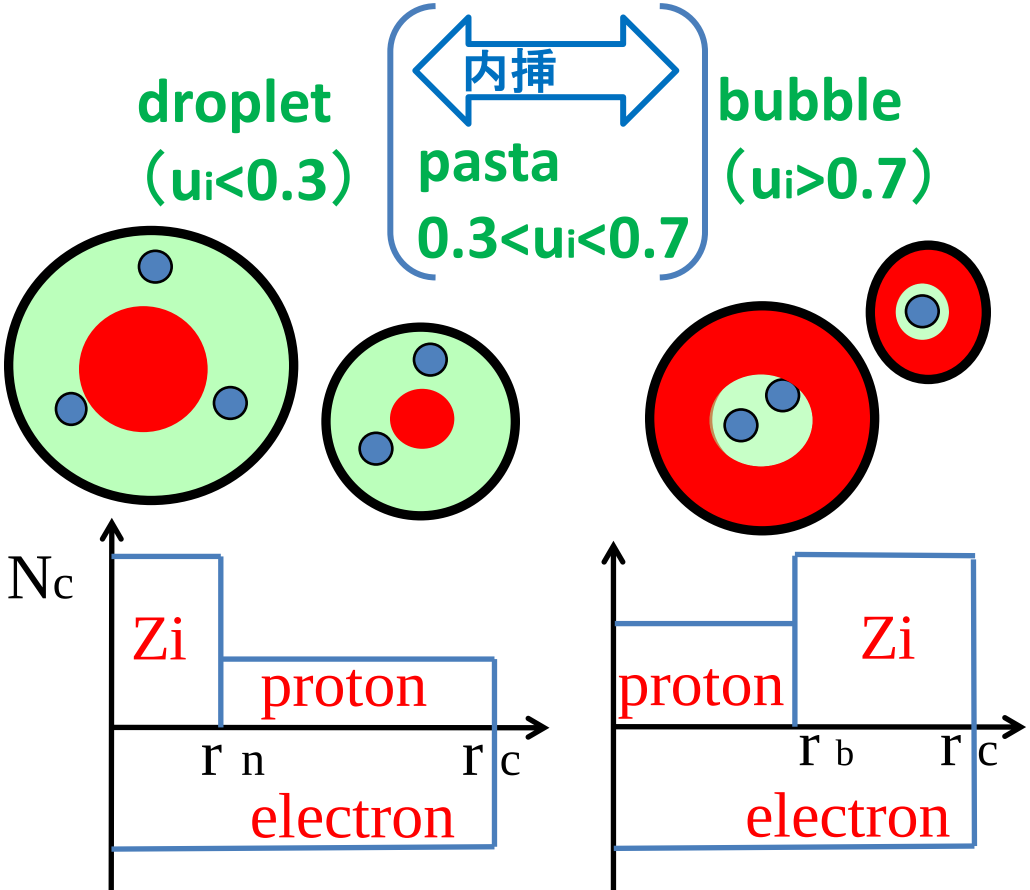
$$\boxed{V_i n_e = Z_i + (V_i - V_n) n'_p}$$

セルの体積 V^i

原子核の体積 V_n^i

気相の体積 V_b^i

$$\boxed{u^i = \frac{V_n^i}{V^i}}$$



droplet
（ui<0.3）

内挿

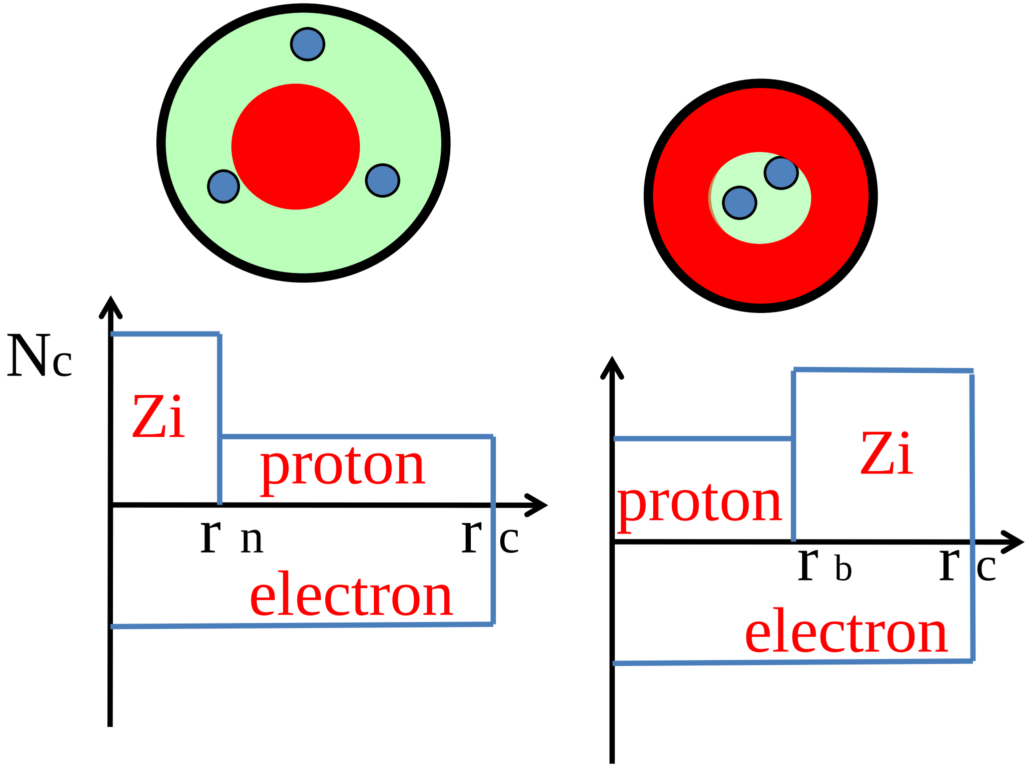
pasta
0.3<ui<0.7

bubble
（ui>0.7）

③クーロンエネルギー

$$M_i = E_i^B + E_i^S + \boxed{E_i^C}$$

E_i^{coul} ＝Cell内のクーロン力を積分



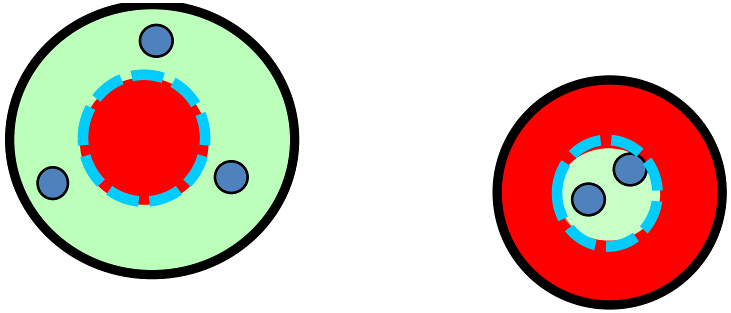
$$E_i^{coul} = \begin{cases} \frac{3}{5}\left(\frac{3}{4\pi}\right)^{-1/3}\frac{e^2}{n_s^2}\left(\frac{Z_i - \boxed{n'_p V_n^i}}{A_i}\right)^2 V_n^{i\,5/3}\,\boxed{D(u_i)} & (u_i < 0.3) \\ \frac{3}{5}\left(\frac{3}{4\pi}\right)^{-1/3}\frac{e^2}{n_s^2}\left(\frac{Z_i - \boxed{n'_p V_n^i}}{A_i}\right)^2 V_b^{i\,5/3}\,\boxed{D(1-u_i)} & (u_i > 0.7) \end{cases}$$

$$D(u_i) = \left(1 - \frac{3}{2}u_i^{1/3} + \frac{1}{2}u_i\right)$$

クーロン補正

④表面エネルギー

$$M_i = E_i^B + E_i^S + E_i^C$$



E_i^{surf}

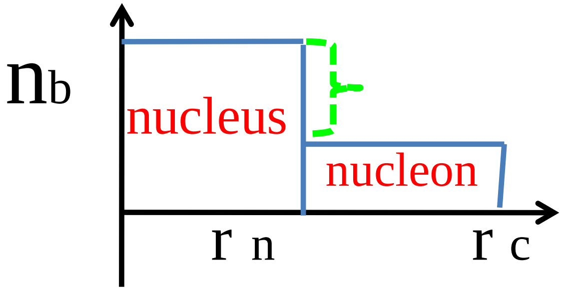
＝表面積×表面張力σ

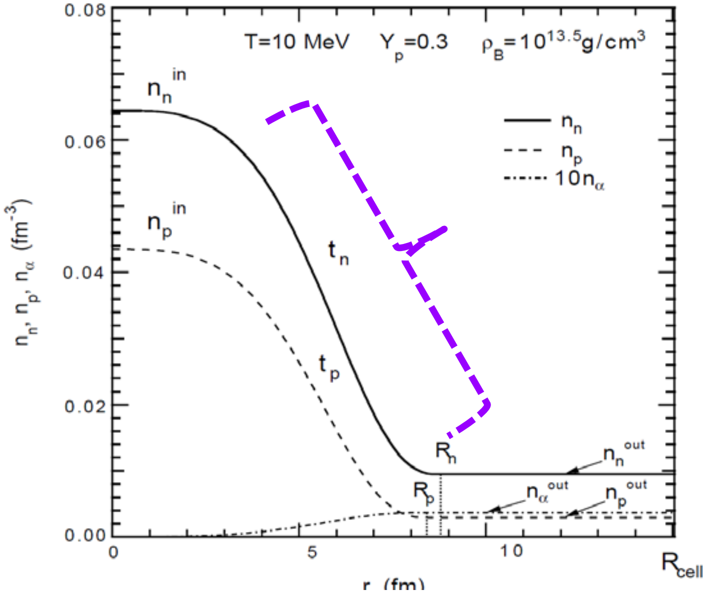
・表面対称エネルギー

（σのZ/A依存性　Lattimer 1991）

・気相の核子から受ける核力の補正

(　Shen EOSでは$E_i^{surf} \propto \int dr\, |\nabla n_b(r)|^2$　)





$$E_i^S = \begin{cases} 4\pi {r_i^N}^2 \sigma_i \left(1 - \frac{n'_p + n'_n}{n_{si}}\right)^2 = 4\pi \left(\frac{3}{4\pi} V_i^N\right)^{2/3} \sigma_i \left(1 - \frac{n'_p + n'_n}{n_{si}}\right)^2 & (u_i < 0.3) \\ 4\pi {r_i^B}^2 \sigma_i \left(1 - \frac{n'_p + n'_n}{n_{si}}\right)^2 = 4\pi \left(\frac{3}{4\pi} V_i^B\right)^{2/3} \sigma_i \left(1 - \frac{n'_p + n'_n}{n_{si}}\right)^2 & (u_i > 0.7) \end{cases}$$

⑤体積、対称エネルギー

$$M_i = E_i^B + E_i^S + E_i^C$$

低密度側

質量が**実験値**になるように定義（shell energy（magic numberなど）を取り入れる）
（実験値から密度0におけるクーロン、表面エネを引いておく）

$$\boxed{E_i^{bulk} = M_i^{\mathrm{data}} - [E_i^{coul}]_{\rho,n'_p,n'_n=0} - [E_i^{surf}]_{\rho,n'_p,n'_n=0}}$$

高密度側　及び　実験値のない原子核

原子核内部は核密度 ⇒ 核密度 $Y_p = Z_i/A_i$ で**RMF**計算

$$\boxed{E_i^{bulk}(T) = A_i \underline{F^{RMF}(n_{si}, T, Z_i/A_i)}}$$

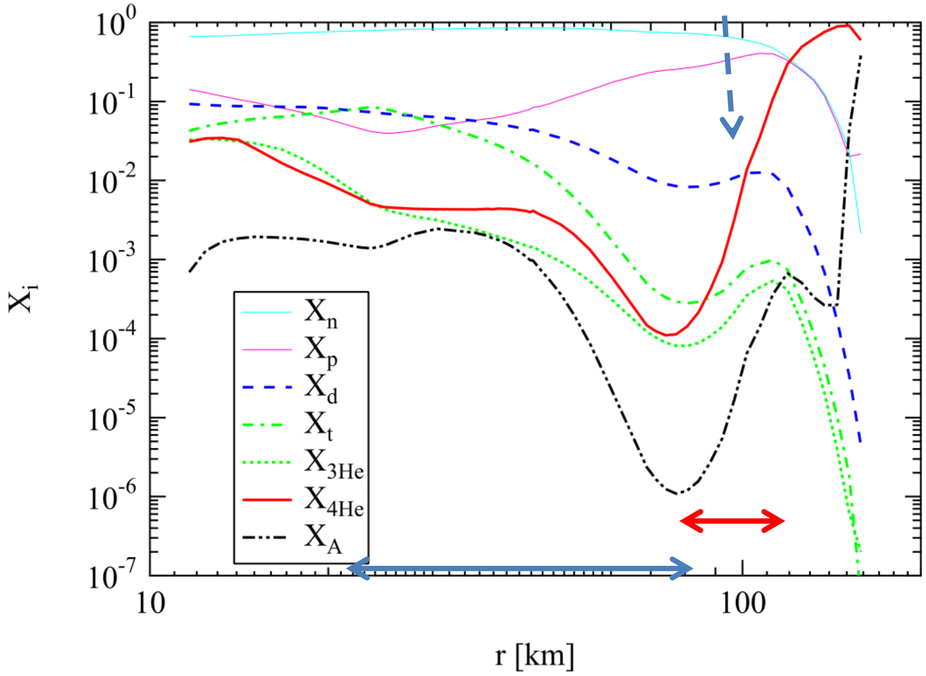
Free Energy per baryon

その間を密度の関数で内挿する

Shen

●バウンス後　軽元素の加熱冷却への影響（軽元素の分布）

15太陽質量　バウンス150ms後の分布



p,nだけではなく、d、Heもそこそこいる。

Heating region ではdeuteronが１%程度
Cooling　regionでは、Heよりも多い

K. Sumiyoshi and G. Röpke, (2008)

核子とニュートリノの反応断面積

（R=100km，Lν＝10^52erg/sec，Tν=5MeV　）

nucleon	deuteron
223.1	55.6

[MeV/sec/nucleon]

deuteronによる25%ほど効く

S.Nakamura et al. (2009)

ア内の軽元素（deuteron）がHeatingやCoolingに効くかもしれない？

⑥一様核物質の遷移

Case1: 系のYpが0.5に近い場合（ Yp=0.3,0.5）

- Z/A~Ypの原子核が多い
- bubbleが小さくなりくっつく

原子核セル内における陽子、中性子の密度分布の遷移

(Z=6,A=20,Yp=0.3,T=10MeV)

------- セル境界

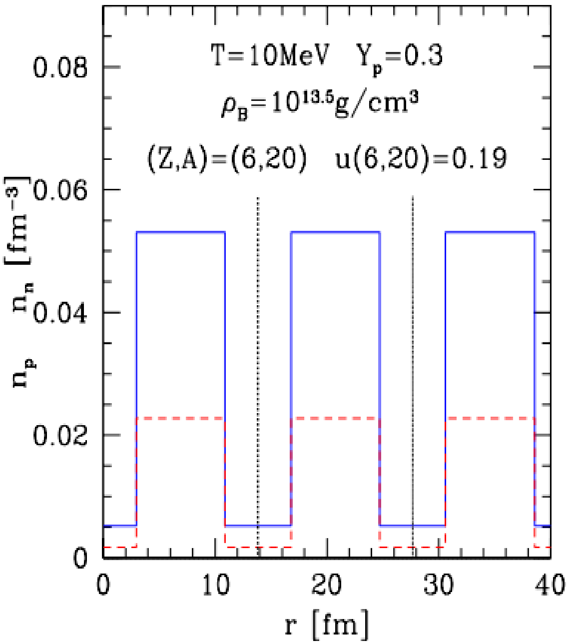
------- 陽子

——— 中性子

$\rho = 10^{13.5}\mathrm{g/cm^3}$

u=0.19

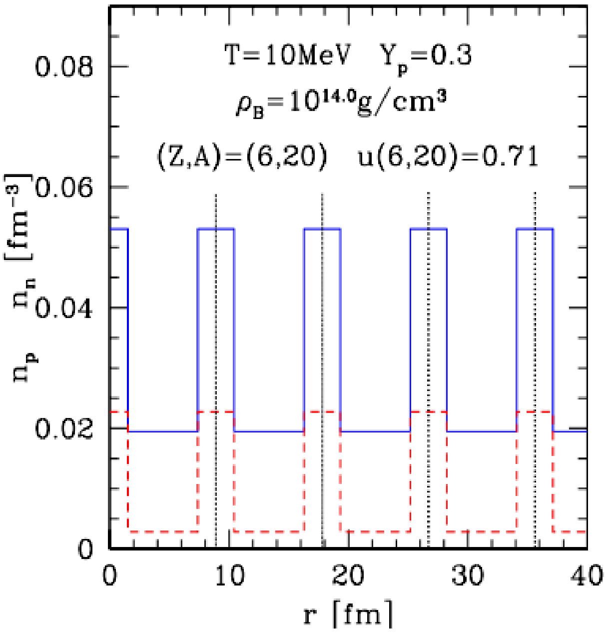
(droplet)



$\rho = 10^{14.0}\mathrm{g/cm^3}$

u=0.71

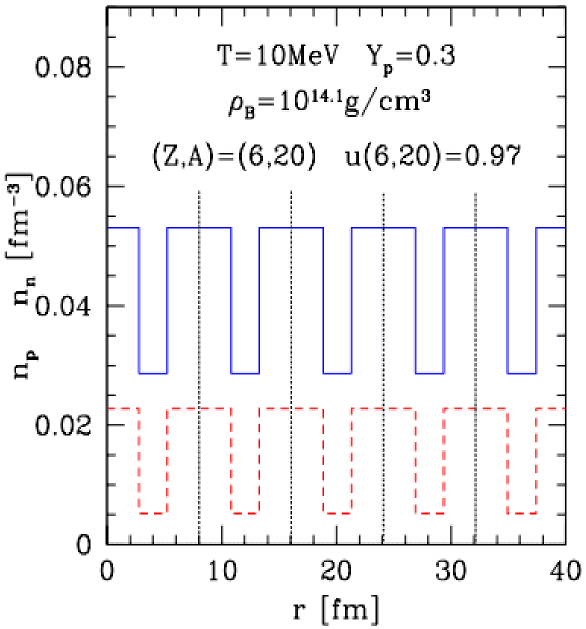
(bubble)



$\rho = 10^{14.1}\mathrm{g/cm^3}$

u=0.97

(bubble)



⑥一様核物質の遷移

Case2: 系のYpが0.1に近い場合（ Yp=0.1)

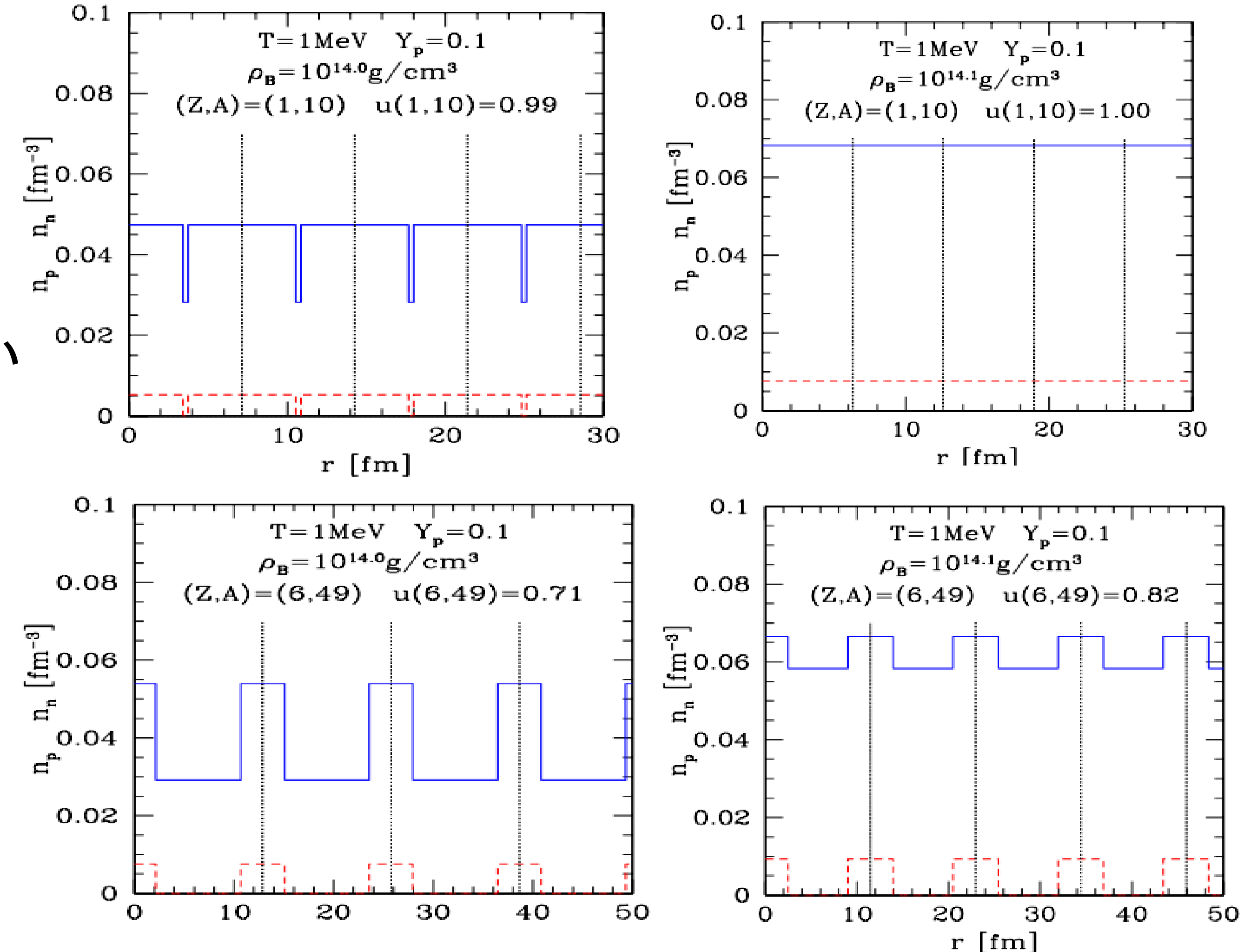
・Z/A=Ypの原子核①(Z=1,A=10)は一様核物質
・ Z/A＞Ypの原子核②（Z=6,A=49）はバブル
最終的には①が増えてuniform

$\rho = 10^{14.0}\mathrm{g/cm^3}$ $\rho = 10^{14.1}\mathrm{g/cm^3}$

①(Z=1,A=10)

①は対称エネが大きい
非一様になってでも
②がいた方が良い

②（Z=6,A=49）



陽子、中性子、軽原子核(Z<5) 、重原子核(Z>6) のMass Fraction

Our Model Shen