格子QCDによるS=−2バリオン間相互作用のクォーク質量依存性的研究

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Introduction
Baryon-baryon interactions are crucial for nuclear and astrophysics

For NN, large amount of scattering data
For YN and YY, experimental data are scarce.

Meson exchange model
Described by hadron dof with phenomenological repul. core

Repulsive core is generated by $\omega$ meson exchange.

Nucleus

Hypernucleus

Quark cluster model
Effective meson ex + quark anti-symmetrization
Quark Pauli effects are taking into account
Introduction

Study the hyperon-nucleon (YN) and hyperon-hyperon (YY) interactions

Strangeness brought the deeper understanding of BB interaction.

Three flavor (u,d,s) world

H-dibaryon state is expected

Wide variety of BB interaction

Flavor symmetric

Flavor anti-symmetric

SU(3) breaking

I=0 states
1, 8s, 27 mixing

I=1 states
8s, 27 mixing

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**Introduction**

- **S=-2 BB interaction**
  - Direct scattering experiment is very difficult.
  - All irreducible representations are involved.

- **H-dibaryon**
  - Flavor singlet object (6q / tightly bound)
  - No Pauli blocking was proved by quark cluster model.
  - Strong color magnetic (attractive) interaction

**Recent Lattice QCD studies**

- **HAL QCD: SU(3) limit**
  - $BE = 26\text{MeV} \quad m_{\pi} = 470\text{MeV}$

- **NPLQCD: SU(3) breaking**
  - $BE = 13\text{MeV} \quad m_{\pi} = 390\text{MeV}$

**Conclusions of the “NAGARA Event”**

K.Nakazawa and KEK-E176 & E373 collaborators

- $\Lambda-N$ attraction
- $\Lambda-\Lambda$ weak attraction
  - $m_H \geq 2m_\Lambda - 6.9\text{MeV}$

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HAL QCD strategy
QCD to hadronic interactions

**QCD Lagrangian**

\[ L_{QCD} = \bar{q} \left( i \gamma_\mu D^\mu - m \right) q + \frac{1}{4} F^{a\mu\nu}_\mu F^{a\mu\nu}_\nu \]

**Lattice QCD simulation**

The potential through our method reproduce to the phase shift by QCD

The potential through our method reproduce to the phase shift by QCD

**HAL QCD method**


Lattice QCD simulation can connect the fundamental QCD with nuclear physics
**Nambu-Bethe-Salpeter wave function**

**Definition:** equal time NBS w.f.

\[
\Psi_\nu(E, t - t_0, \vec{r}) = \sum_{\vec{x}} \langle 0 | B_i(t, \vec{x} + \vec{r}) B_j(t, \vec{x}) | E, \nu, t_0 \rangle 
\]

\[B = \epsilon_{abc} (q_a C \gamma_5 q_b) q_c\]

The ket stands for the eigenstate of the complete set of observables

\[
E : \text{Total energy of system} \\
\nu : \text{other observables which needs to form the complete set}
\]

Local composite interpolating operators

\[
p_\alpha = \epsilon_{c_1 c_2 c_3} (C \gamma_5)_{d_1 d_2} \delta_{d_3 \alpha} u(\xi_1) d(\xi_2) u(\xi_3)
\]

\[
\Sigma^0_\alpha = -\epsilon_{c_1 c_2 c_3} (C \gamma_5)_{d_1 d_2} \delta_{d_3 \alpha} \sqrt{\frac{1}{2}} \left[ d(\xi_1) s(\xi_2) u(\xi_3) + u(\xi_1) s(\xi_2) d(\xi_3) \right]
\]

\[
\Lambda_\alpha = -\epsilon_{c_1 c_2 c_3} (C \gamma_5)_{d_1 d_2} \delta_{d_3 \alpha} \sqrt{\frac{1}{6}} \left[ d(\xi_1) s(\xi_2) u(\xi_3) + s(\xi_1) u(\xi_2) d(\xi_3) - 2 u(\xi_1) d(\xi_2) s(\xi_3) \right]
\]

NBS wave function has a same asymptotic form with quantum mechanics.

(NBS wave function is characterized from phase shift)

\[
\Psi(t - t_0, \vec{r}) \approx A \frac{\sin(pr + \delta(E))}{pr}
\]
Define the energy-independent potential in Schrödinger equation (most general form)

\[
\left(\frac{k^2}{2\mu} - H_0\right)\Psi(\vec{x}) = \int U(\vec{x}, \vec{y}) \Psi(\vec{y}) d^3 y
\]

Recent development: Time dependent method.

We replace \(\psi\) to \(R\) defined below

\[
\partial_t R_{\alpha}(\vec{x}, E) \equiv \partial_t \left( A \frac{\Psi_{\alpha}(\vec{x}, E) e^{-Et}}{e^{-m_s t} e^{-m_q t}} \right) \propto -\frac{p_{\alpha}^2}{2\mu_{\alpha}} R_{\alpha}(\vec{x}, E)
\]

Performing the derivative expansion for the interaction kernel

\[
\left(-\frac{\partial}{\partial t} - H_0\right) R(\vec{x}) = \int U(\vec{x}, \vec{y}) R(\vec{y}) d^3 y
\]

Taking the leading order of derivative expansion of non-local potential

\[
U(\vec{x}, \vec{y}) \simeq V_0(\vec{x}) \delta(\vec{x} - \vec{y}) + V_1(\vec{x}, \nabla) \delta(\vec{x} - \vec{y}) \ldots
\]

Finally local potential was obtained as

\[
V(\vec{x}) = -\frac{\partial_t R(\vec{r})}{R(\vec{r})} + \frac{1}{2\mu} \frac{\nabla^2 R(\vec{x})}{R(\vec{x})}
\]
**Coupled channel Schrödinger equation**

**Preparation for the NBS wave function**

\[ \Psi^\alpha(E, t, \vec{r}) = \sum_{\vec{x}} \langle 0 | (B_1 B_2)^\alpha(t, \vec{x}) | E \rangle \]

\[ \Psi^\beta(E, t, \vec{r}) = \sum_{\vec{x}} \langle 0 | (B_1 B_2)^\beta(t, \vec{x}) | E \rangle \]

**Inside the interaction range**

In the leading order of velocity expansion of non-local potential,

\[
\left( \frac{p_\alpha^2}{2 \mu_\alpha} + \nabla^2 \right) \psi^\alpha(\vec{x}, E) = V^\alpha(\vec{x}) \psi^\alpha(\vec{x}, E) + V^\alpha(\vec{x}) \psi^\beta(\vec{x}, E)
\]

**Two-channel coupling case**

The same “in” state

Factorization of interaction kernel

\[ \mu_\alpha : \text{reduced mass} \]

\[ p_\alpha : \text{asymptotic momentum.} \]

**Factorization of interaction kernel**

Asymptotic momentum are replaced by the time-derivative of \( R \).

\[ R^{B_1 B_2}(t, \vec{r}) = \sum_{\vec{x}} \langle 0 | B_1(t, \vec{x}+\vec{r}) B_2(t, \vec{x}) | I(0) | 0 \rangle e^{(m_1+m_2)t} \]

\[
\left( \begin{array}{cc}
V^\alpha(\vec{r}) & V^\beta(\vec{r}) \\
V^\alpha(\vec{r}) & V^\beta(\vec{r})
\end{array} \right) = \left( \begin{array}{cc}
\left( \frac{\nabla^2}{2 \mu_\alpha} - \frac{\partial}{\partial t} \right) R^\alpha_{II}(\vec{r}, E) & \left( \frac{\nabla^2}{2 \mu_\beta} - \frac{\partial}{\partial t} \right) R^\beta_{II}(\vec{r}, E) \\
\left( \frac{\nabla^2}{2 \mu_\alpha} - \frac{\partial}{\partial t} \right) R^\alpha_{I2}(\vec{r}, E) & \left( \frac{\nabla^2}{2 \mu_\beta} - \frac{\partial}{\partial t} \right) R^\beta_{I2}(\vec{r}, E)
\end{array} \right) \left( \begin{array}{cc}
R^\alpha_{II}(\vec{r}, E) & R^\beta_{II}(\vec{r}, E) \\
R^\alpha_{I2}(\vec{r}, E) & R^\beta_{I2}(\vec{r}, E)
\end{array} \right)^{-1}
\]

\[
x = \frac{\exp(-m_\alpha t)}{\exp(-m_\beta t)}
\]
Results
2+1 flavor gauge configurations by PACS-CS collaboration.
- RG improved gauge action & O(a) improved clover quark action
- $\beta = 1.90$, $a^{-1} = 2.176$ [GeV], $32^3 \times 64$ lattice, $L = 2.902$ [fm].
- $\kappa_s = 0.13640$ is fixed, $\kappa_{ud} = 0.13700, 0.13727$ and $0.13754$ are chosen.

Flat wall source is considered to produce S-wave B-B state.

The KEK computer system A resources are used.

<table>
<thead>
<tr>
<th>In unit of MeV</th>
<th>$E_{sb, 1}$</th>
<th>$E_{sb, 2}$</th>
<th>$E_{sb, 3}$</th>
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<tbody>
<tr>
<td>$\pi$</td>
<td>701±1</td>
<td>570±2</td>
<td>411±2</td>
</tr>
<tr>
<td>$K$</td>
<td>789±1</td>
<td>713±2</td>
<td>635±2</td>
</tr>
<tr>
<td>$m_{\pi}/m_K$</td>
<td>0.89</td>
<td>0.80</td>
<td>0.65</td>
</tr>
<tr>
<td>$N$</td>
<td>1585±5</td>
<td>1411±12</td>
<td>1215±12</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>1644±5</td>
<td>1504±10</td>
<td>1351±8</td>
</tr>
<tr>
<td>$\Sigma$</td>
<td>1660±4</td>
<td>1531±11</td>
<td>1400±10</td>
</tr>
<tr>
<td>$\Xi$</td>
<td>1710±5</td>
<td>1610±9</td>
<td>1503±7</td>
</tr>
</tbody>
</table>

$E_s$: 2702MeV
$E_s^o$: 2718MeV
$E_s^o$: 2800MeV

$\Lambda\Lambda$ : 3288MeV
$N\Xi$ : 3295MeV
$\Sigma\Sigma$ : 3320MeV

$E_s^o$: 2702MeV
$E_s^o$: 2718MeV
$E_s^o$: 2800MeV

3008MeV
3021MeV
3062MeV

SU(3) breaking effects becomes larger

u,d quark masses lighter
In this channel, our group found the “H-dibaryon” in the SU(3) limit.

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Comparison of potential matrices

Transformation of potentials from the particle basis to the SU(3) irreducible representation (IR) basis.

SU(3) Clebsh-Gordan coefficients

\[
\begin{pmatrix}
1 \\
8 \\
27
\end{pmatrix}
= U
\begin{pmatrix}
\Lambda \Lambda \\
N \Xi \\
\Sigma \Sigma
\end{pmatrix},
\]

\[
U
= 
\begin{pmatrix}
V^{\Lambda \Lambda} & V^{\Lambda \Lambda} & V^{\Lambda \Lambda} \\
V^{N \Xi}_{\Lambda \Lambda} & V^{N \Xi}_{N \Xi} & V^{N \Xi}_{\Sigma \Sigma} \\
V^{\Sigma \Sigma}_{\Lambda \Lambda} & V^{\Sigma \Sigma}_{N \Xi} & V^{\Sigma \Sigma}_{\Sigma \Sigma}
\end{pmatrix}
U^t \rightarrow 
\begin{pmatrix}
V_1 \\
V_8 \\
V_{27}
\end{pmatrix}
\]

In the SU(3) irreducible representation basis, the potential matrix should be diagonal in the SU(3) symmetric configuration.

Off-diagonal part of the potential matrix in the SU(3) IR basis would be an effectual measure of the SU(3) breaking effect.

We will see how the SU(3) symmetry of potential will be broken by changing the u,d quark masses lighter.

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1, 8_s, 27 (l=0) \(^1S_0\) channel

**Diagonal elements**

![Graphs showing diagonal elements](image1)

- Strongly attractive
  - H-dibaryon channel

**Off-diagonal elements**

![Graphs showing off-diagonal elements](image2)

- Pauli blocking effect

Mixture of singlet and octet
- Is relatively larger than the others
- 27 plet does not mix so much to the other representations

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**Esb1**: \(m_\pi = 701\) MeV
**Esb2**: \(m_\pi = 570\) MeV
**Esb3**: \(m_\pi = 411\) MeV
$\Lambda\Lambda$ and $N\Xi$ phase shifts

- **Esb1**: 
  - Bound H-dibaryon
- **Esb2**: 
  - H-dibaryon is near the $\Lambda\Lambda$ threshold
- **Esb3**: 
  - The H-dibaryon resonance energy is close to $N\Xi$ threshold.

We can see the clear resonance shape in $\Lambda\Lambda$ phase shifts for Esb2 and 3.

The “binding energy” of H-dibaryon from $N\Xi$ threshold becomes smaller as decreasing of quark masses.

Preliminary!
We have investigated the S=-2 BB system from lattice QCD.

In order to deal with a variety of interactions, we extend our method to the coupled channel formalism.

Potentials are derived from NBS wave functions calculated with PACS-CS configurations.

Quark mass dependence of potentials can be seen not in long range region but in short distances as an enhancement of repulsive core.

Small mixture between different SU(3) IRs can be seen as the flavor SU(3) breaking effect.

SU(3) breaking effects are still small even in $m_\pi/m_K=0.65$ situation but it would be change drastically at physical situation $m_\pi/m_K=0.28$. 