超新星爆発の物理と数値シミュレーション

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Fundamentals of Core-Collapse Supernovae





Fundamentals of Core-Collapse Supernovae

Triggered by the gravitational collapse of massive stars (> 10M.)

- Pre-explosion
 One of the most energetic phenomena in the Post-explosion Universe
 - E_□ 10⁵³erg, E_{kin} □ 10⁵¹erg, E_□ □ 10⁴⁹erg
- Sites for high energy phenomena and important for chemical evolutions in the universe
 - produce neutrinos, gravitational waves, cosmic rays, X-rays, gamma-rays
 - nucleosynthesis of heavy elements

Challenges in Supernova Research

Supernova is a complex interplay of

■ Micro Physics

- weak interactions
 - neutrino interaction rates with matter
 - neutrino oscillations
- nuclear physics
 - equation of state
 - many body effects on neutrino reaction rates

■ Macro Physics

- hydrodynamics
 - rotation
 - convection
- radiative transport
- general relativity
 - gravitational waves
- magnetic field

We have to treat them all simultaneously and consistently.

Goals in Supernova Research

The supernova theory must address the following issues:



- ✓ How does the explosion occur and are the neutron star mass and explosion energy determined?
- ✓ How do the progenitors correspond to the supernova types?
- ✓ What is the origin of rotation, magnetic field, and proper motion of neutron stars?
- ✓ What is the relationship with other high energy objects such as GRBs
 - hypernovae, magnetars
- ✓ How do syntheses of heavy elements proceed?
 - explosive nucleosynthesis, r-process

CCSNe can be a new probe into the properties of dense hadronic matter as well as neutrinos!

Which Mass Stars Should We Blow Up?

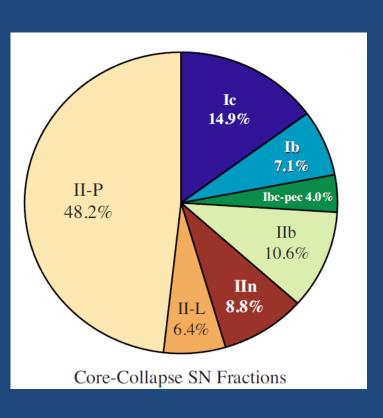
✓ The present universe may be producing stars as massive as ~300M_{solar}.

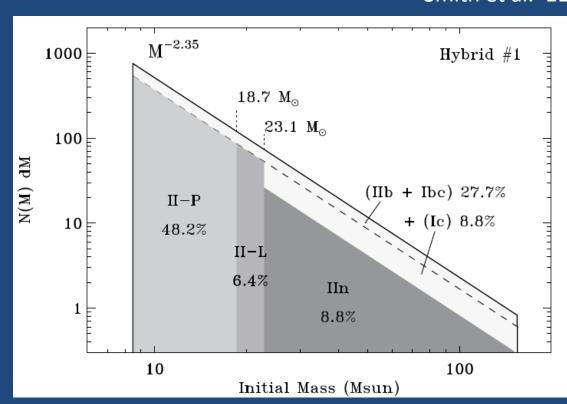
Crowther et al. '10

- ✓ There is observational evidence that NS's are formed from very massive stars.
 - SGR 1806-20: ≥ 50M.
 - anomalous X-ray pulsar in young massive galactic cluster Westerlund 1: ≥ 40M □
 - anomalous X-ray pulsar 1E1048.1-5937
 embedded in stellar wind bubble: □ 30 □ 40M □
- ✓ The best bet for the minimum mass to produce CCSNe is 8±1M_{solar} at present.
- ✓ SNeII-P have been observed to be produced by $8.5-17M_{solar}$ stars.
- ✓ Most of massive stars may explode to produce neutron stars!
 - Core masses are not monotonic owing to mass losses.

SN Fractions

Smith et al. '11





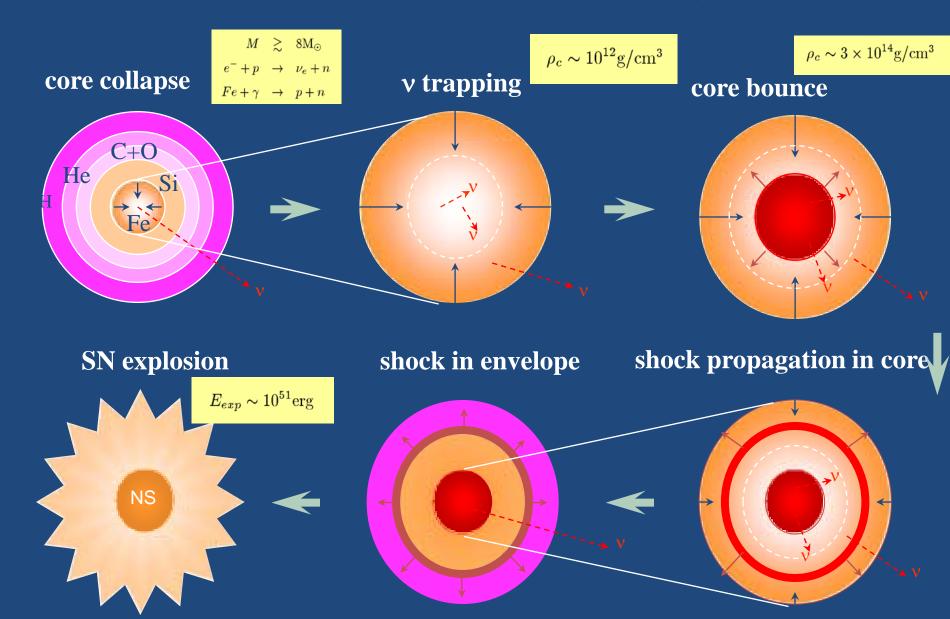
Quiet collapse to BH may not be required!

Very Luminous SNe

- ✓ Some SNe II-L (SN2008es, SN2005ap) and SNe IIn (SN2006gy, SN2006tf) are very bright.
 - E_v ~10⁵¹ erg
 - The late time light curves disfavor Ni-powered brightening.
 - Interactions with CSM produced by LBV-like activities are more likely
 - Explosion mechanisms are unknown.
 - pulsational instability
 - jet explosion as in GRB etc.
- ✓ Most luminous SNe are typically found in faint, small dwarf galaxies.
- ✓ There is another set of luminous SNe that show no H in their spectra, but have unusual light curves and spectra. → SNIpec
- ✓ Two luminous SNe (SN1999as and SN2007bi) show strong evidence of having been produced as a result of the pulsational pair instability. →SNIpp
 - E $_{\rm exp}$ > $10^{52} {\rm erg}$
 - ⁵⁶Ni > 4M_{solar}
 - M $_{init}$ > 150M $_{solar}$
 - There is evidence that SN2007bi was a single star.
 - SNIpec may be also of pulsational pair instability-origin, but not so extreme.

Canonical Evolutions of Core Collapse

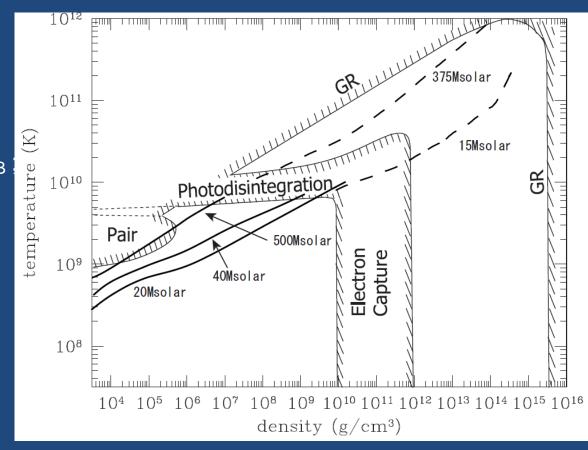
Scenario of Collapse-Driven Supernovae



Onset of Core Collapse

- ✓ Cores of massive stars collapse when they lose stability against radial perturbations.
- > (Newtonian) Criterion:

- ➤ Cores have low entropies, $S \square 1k_B$ and are supported by degenerate electrons just like white dwarfs, having $\square \cdot 4=3$:
- \triangleright Electron captures and photodisintegrations tend to reduce Γ .
- As the density and temperature rise, these reactions proceed further, making the core even more unstable (positive feedback).



Neutrino Trapping

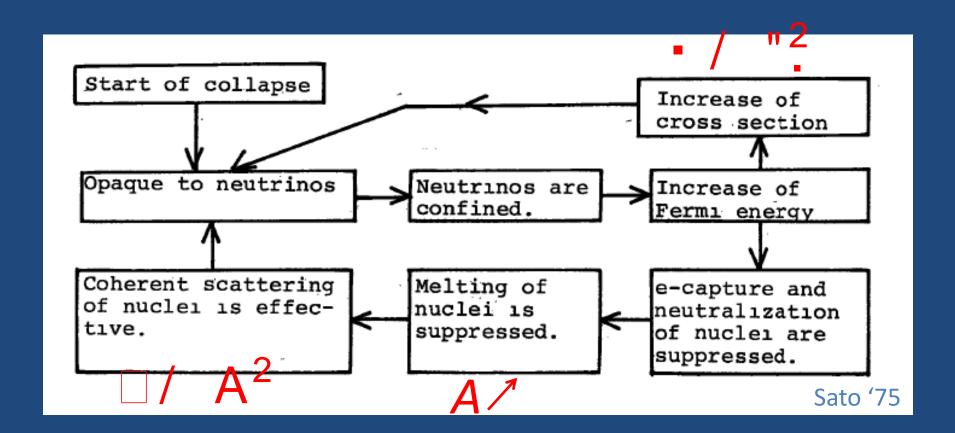
✓ Coherent scatterings on nuclei are the main source of neutrino opacity.

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v wave length: - 20fm \frac{}{10\text{MeV}}
Nuclear radius: \Box 5fm (A=56)<sup>1=3</sup>
v mean free path: \Box 10<sup>6</sup> cm \frac{\Box}{10 \text{MeV}} \frac{\Box}{10^{12} \text{g=cm}^3} \frac{\Box}{10^{12} \text{g=cm}^3}
core radius: \Box 10<sup>7</sup> cm
v Diffusion time scale: \Box 100 msec \frac{R_{PNS}}{3 \Box 10^6 \text{cm}} = \frac{100 \text{ meV}}{10 \text{ MeV}} = \frac{10^{14} \text{ g}}{10^{14} \text{ g}}
Dynamical time scale: \Box 10 msec \frac{\Box}{10^{12}\text{g=cm}^3}! \Box 10 msec \frac{\Box}{10^{12}\text{g=cm}^3}
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- ✓ Neutrinos are essentially trapped in the core at $\rho \sim 10^{11} \text{g/cm}^3$ and diffuse out of the core thereafter.
- \checkmark β-equilibrium is established at $\rho \sim 10^{12}$ g/cm³ and dynamics becomes adiabatic from this point to shock formation.

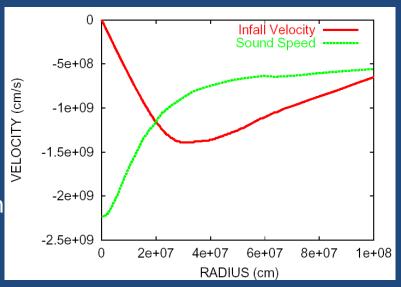
Positive Feedback in v-trapping

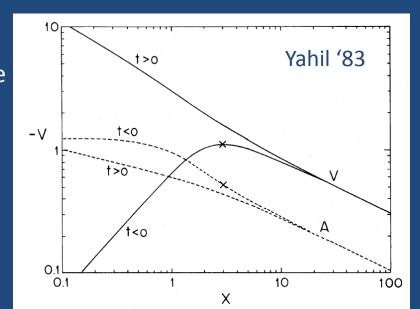
- ✓ Weak neutral currents predicted by W-S theory have profound implications for supernova theory
 - Coherent scatterings make a core opaque for neutrinos.
 - Neutronization occurs much more slowly than the dynamical time scale.
 - Neutrinos diffuse out of the core.



Homologous Collapse

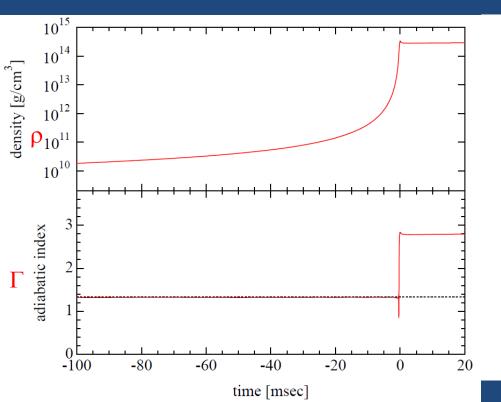
- ✓ Infalling cores are divided into two parts :
 - inner core: subsonic and homologous
 - outer core: supersonic and free fall-like
- ✓ The outer core is causally disconnected from the inner core.
 - A shock wave produced at the boundary between the inner and outer cores.
- ✓ Yahil found a self-similar solution for Γ close to but slightly smaller than 4/3.
 - The inner core mass is close to the Chandrasekhar mass corresponding to the reduced electron fraction and decreases in time.

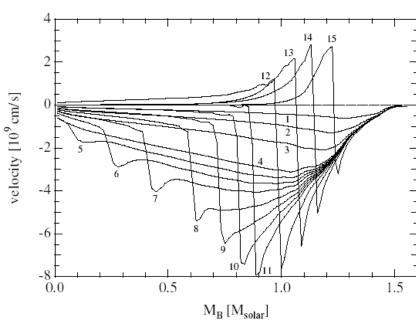




Core Bounce

- ✓ Matter becomes very stiff when the nuclear saturation density is exceeded.
- ✓ Matter then recovers stability against collapse and the inner core, the interior of which is causally connected, bounces as a whole.





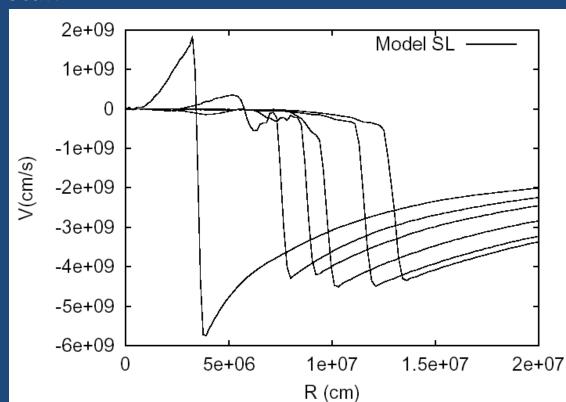
✓ Most of gravitational energy is compensated for by internal energy and the energy imparted to the kinetic energy of outward motions is ~ a few x 10⁵¹ erg.

Energy scales of relevance

- Rest mass energy: 10^{54} erg $\frac{M}{M}$.
- - Neutron star: $\square \ \square \ 10^{53} \text{erg} \ \frac{\text{M}}{\text{M}_{\square}} \ \frac{\text{R}_{\text{N S}}}{10^{6} \text{cm}}$
- \triangleright Shock wave: a few \square 10⁵¹erg

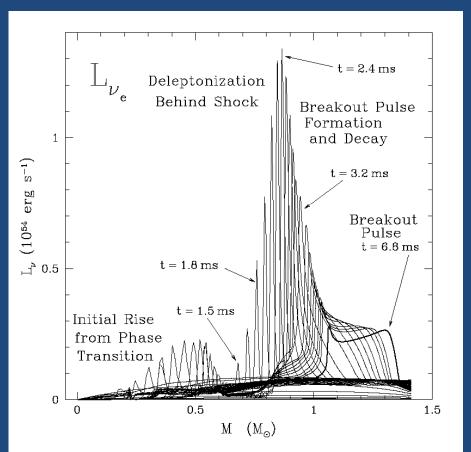
Shock Propagation & Stagnation

- ✓ The shock wave generated by core bounce propagates outwards initially.
- ✓ Photo-disintegrations of nuclei and neutrino cooling are energy sinks that make the shock wave stagnated at ~200km from the center in the core.
- ✓ The standing accretion shock wave then starts to recede back onto a nascent proto neutron star.
- ➤ Temperatures behind the shock wave is ~1MeV, high enough to dissociate heavy nuclei to Helium and nucleons.
- About 10⁵¹erg is consumed to dissociate every 0.1M_{solar} of heavy nuclei.
- Massive cores and/or small inner cores are disadvantageous for successful shock break-out of the core.



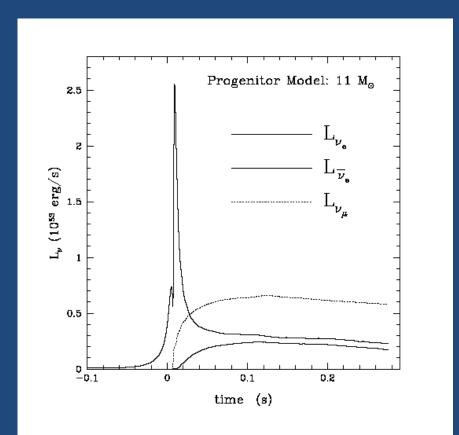
Neutronization Bursts

✓ When the shock wave reaches the v-sphere, heavy nuclei are photo-dissociated and the opacity is suddenly reduced and electron captures on protons are enhanced considerably, leading to the neutronization burst.

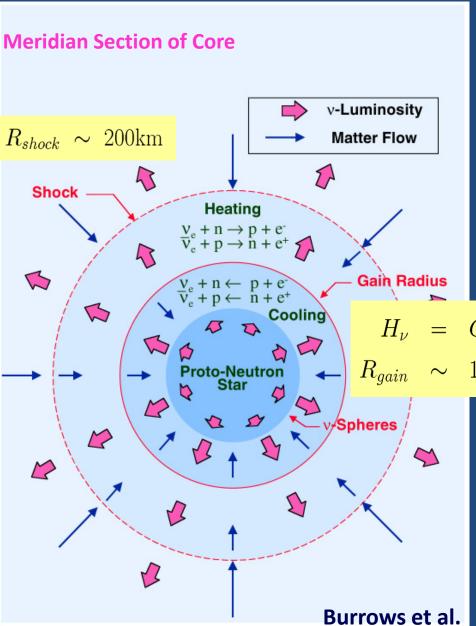


Neutronization Bursts

- ✓ Up to core bounce only v_e 's are produced and accumulated in the core.
- ✓ The luminosity is high, O(10⁵³erg/s) but the total energy is small, O(10⁵¹erg). The burst will be detectable for SuperKamiokande for a Galactic event.



Neutrino Heating & Shock Revival



✓ Most of the liberated gravitational energy is stored in the proto neutron star as internal energy, which can be tapped by neutrinos.

$$E_G \cdot E_{int} \square 10^{53} erg$$

✓ The initial shock energy is not large enough to push through the outer core.

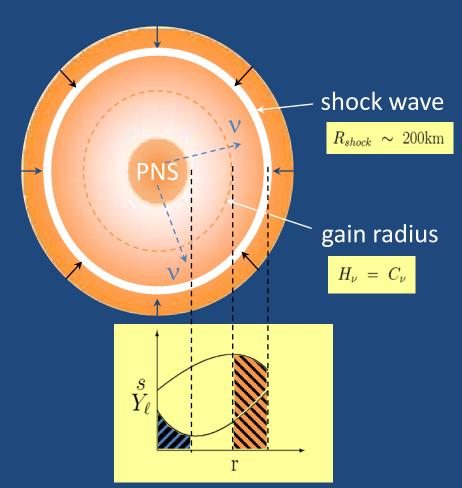
$$E_{sh} \stackrel{<}{\subseteq} 5 \square 10^{51} erg < E_{\square;Fe}^{loss}$$

- ✓ The shock is stalled inside the core
 and becomes an accretion shock.

 The shock should be somehow revived
- ✓ The spherically symmetric configuration is unstable!

Hydrodynamical Instabilities

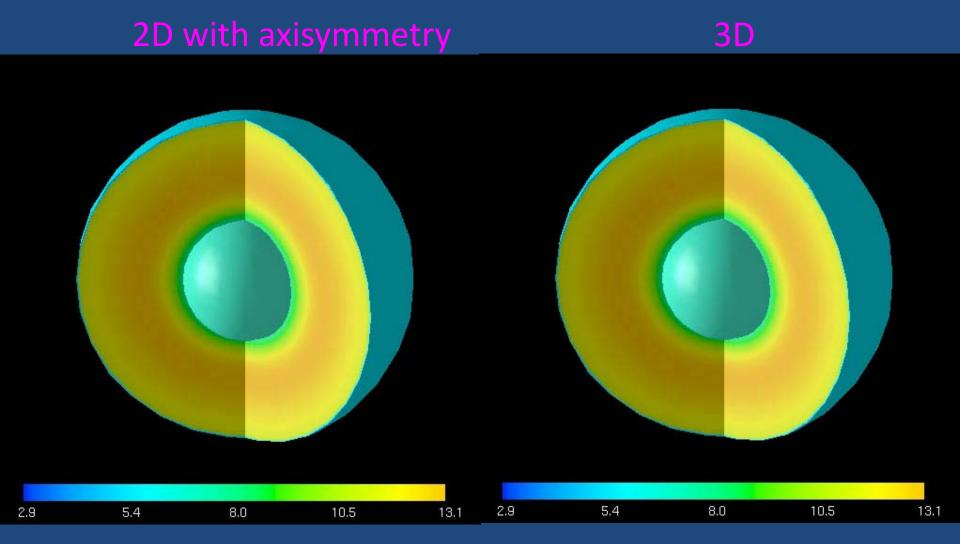
✓ Post bounce configuration



✓ Convections

- prompt convection
 - weakening of prompt shock
 - entropy-driven
 - just behind shock
 - not sustained long
- Bethe convection
 - neutrino heating
 - entropy-driven
 - between shock and gain radius
- <u>lepton-driven convection</u>
 - neutrino diffusion
 - lepton-driven
 - around ν-sphere
- ✓ Standing Accretion Shock Instability (SASI)
 - acousto-vortex cycle-driven
 - between shock and PNS surface
- √ (g-mode oscillations of PNS)

SASI in 2D and 3D



Summary of CCSNe

- ✓ CCSN is an explosion of massive stars triggered by core collapse.
- √The liberated gravitational energy ~10⁵³erg is far greater than the typical explosion energy ~10⁵¹erg. The problem is that it is stored as internal energy initially and unavailable for explosion directly.
- ✓ ✓'s are the only agents of non-local energy transport that can tap the internal energy stored in PNS.
- √ The density changes from ~10¹¹ to 3-5 x 10¹⁴ g/cm³
 and temperature varies from ~10¹¹ to a few x 10¹¹K.
- ✓ The matter changes its nature drastically at nuclear saturation density ~3x10¹⁴g/cm³.

Summary of CCSNe

- ✓ CCSN is an explosion of massive stars triggered by core collapse.
- √ The liberated gravitational energy ~10⁵³erg is far greater

We are required to compute gas dynamics with the energy-transport by neutrinos and thermodynamical nature of matter being taken into account properly.

- √ v's are the only agents of non-local energy transport that can tap the internal energy stored in PNS.
- √ The density changes from ~10¹¹ to 3-5 x 10¹⁴ g/cm³
 and temperature varies from ~10¹¹ to a few x 10¹¹K.
- ✓ The matter changes its nature drastically at nuclear saturation density ~3x10¹⁴g/cm³.

Basic Equations & Input Physics

v-Radiation (Magneto) Hydrodynamics

- ✓ Gas dynamics is described with (magneto-) hydrodynamical equations.
 - Dissipations can be neglected, since particle mean free paths are very short.

• •
$$10^{10}$$
 cm $\frac{10^{10}}{10^{10}}$ cm $\frac{10^{10}}{10^{10}}$ cm³

- Variables to be solved are baryonic number density, n_p , electron fraction, Y_c , (= proton fraction, Y_p), entropy per baryon, S, (alternatively, temperature, T, or internal energy density, E, etc.) and velocities, V, (plus magnetic fields, B).
- Continuity eq., Euler eqs., eq. for electron fraction (plus induction eq.) are solved, employing an appropriate EOS.
- Spacetime geometry (gravitational potential in Newtonian gravity) is solved simultaneously.

✓ Continuity Eq.

$$\mathbf{r} \cdot (\mathbf{n}_{\mathsf{B}} \mathbf{u}^{\square}) = \mathbf{0}$$
 : Baryon number conservation

$$r_{\square}(n_{L}u^{\square}) = 0 : Lepton number conservation$$

$$\rightarrow r_{\square}(n_{e}u^{\square}) = \square r_{\square}(n_{e}u^{\square})$$

$$\rightarrow u^{\square}r_{\square}Y_{e} = \square u^{\square}r_{\square}Y_{e}$$

Expressed by collision terms of Boltzmann eq.

* Heavy leptons are not abundant in supernova cores.

Energy-momentum tensors

$$T'' = T_{M}'' + T_{R}'' (+ T_{EM}')$$
 $T_{M}'' = u''u'' + (g''' + u''u'')p$
 $T_{R}'' = E_{R}u''u'' + u''F_{R}' + F''u_{R}' + P_{R}'''$
 $T_{EM}'' = \frac{1}{4}F''F_{C}'' + \frac{1}{4}g'''F_{C}'''F_{C}'''$

Euler Equations

$$r _{\square}T^{\square} = 0$$
or $r _{\square}T^{\square}_{M} = \square r _{\square}T^{\square}_{R} (\square r _{\square}T^{\square}_{EM})$

Expressed by collision terms of Boltzmann eq.

✓ Einstein eq.

$$G^{\cdot \cdot} = T^{\sqcup \sqcup}$$

✓ Equation of State (EOS)

$$p = p(n_B; "; Y_e)$$

- \times ε can be replaced by other thermodynaical variable such as s, T, etc.
- ✓ Induction eq.

$$dF = 0$$

$$r _{\square}F = \frac{4}{c}j^{\square}$$

$$F = 0$$

- ✓ The agents of radiative transport of energy and momentum are neutrinos.
 - Wave lengths of neutrinos are much shorter than the hydrodynamical length scale and neutrinos can be treated as particles and be described by kinetic equations such as Boltzmann equation.
 - Neutrino oscillations are the only processes, in which wave characters of neutrinos manifest themselves in macroscopic phenomena.
 - Mean free paths of neutrinos are longer than the hydrodynamic length scale at low densities (•_B ≤ 10¹¹g=cm³).
 - Neutrino distributions are not the Fermi-Dirac distributions even locally at low densities and should be solved with the kinetic equations.

- ✓ Only electron-type neutrinos are produced before core bounce but all six types of neutrinos are abundant after bounce.
- ✓ Unless heavy leptons are produced, there is no difference between μ and τ -neutrinos.
- ✓ The distribution of v_{μ} (v_{τ}) is different from that of \overline{v}_{μ} (\overline{v}_{τ}) in principle. The difference is minor and neglected in practice.
- ✓ Tiny neutrino masses are neglected unless neutrino oscillations are considered.
 - m. \leq 1ev \square E_{\square} \square O(MeV)

✓ Boltzmann Eq.

$$\frac{d}{d} = \frac{d}{d} = \frac{d}$$

Number current & Energy-momentum Tensor

$$\mathbf{n}^{\square} = \frac{d^{3}p}{\mathsf{E}(\mathsf{p})} \, \mathsf{p}^{\square} \mathsf{f}(\mathsf{x};\mathsf{p}); \quad \mathsf{T}^{\square\square} = \frac{d^{3}p}{\mathsf{E}(\mathsf{p})} \, \mathsf{p}^{\square} \mathsf{p}^{\square} \mathsf{f}(\mathsf{x};\mathsf{p})$$

※ In an orthonormal frame

Change of Number & Energy-momentum Densities

$$r \ _{\square} n^{\square} = \begin{array}{c} \stackrel{\checkmark}{=} \frac{d^{3}p}{\mathsf{E}(p)} \ ^{\square} \frac{\Box f \ (x;p)}{\Box \Box} \ _{c}; \quad r \ _{\square} T^{\square\square} = \begin{array}{c} \stackrel{\checkmark}{=} \frac{d^{3}p}{\mathsf{E}(p)} \ p^{\square} \ ^{\square} \frac{\Box f \ (x;p)}{\Box \Box} \ _{c} \end{array}$$

Microphysical inputs in core collapse simulations:

• EOS: various thermodynamical quantities, such as p, T, μ , c_s and nuclear abundance X_A , as functions of 3 independent variables of your choice, e.g. (n_B, ε, Y_e)

• ν interactions :

plugged in the collision term - of Boltzmann eqs.

EOS

Lattimer & Swesty's EOS

✓ Based on a model free energy per baryon with the Skyrme-type parametrization:

$$f(n, Y_p, T) = E(n, Y_p, T)/n - Ts(n, Y_p, T)$$

$$E(n, Y_p, T) = \sum_{t} \frac{\hbar^2 \tau_t}{2 m_t^*} + \left[a + 4b Y_p (1 - Y_p) \right] n^2 + c n^{1+\delta} - Y_p n \Delta,$$

$$s(n, Y_p, T) = \sum_{t} \left(\frac{5 \hbar^2 \tau_t}{6 m_t^* T} - n_t \eta_t \right) / n.$$

- The parameters a, b, c and δ are determined by the properties of zero temperature symmetric nuclear matter at its saturation density: saturation density, binding energy, bulk symmetry energy and bulk incompressibility.

Shen's EOS

- ✓ Relativistic mean field theory
- ✓ Nuclear interactions are described by meson exchanges.

$$\mathcal{L}_{RMF} = \bar{\psi} \left[i \gamma_{\mu} \partial^{\mu} - M - g_{\sigma} \sigma - g_{\omega} \gamma_{\mu} \omega^{\mu} - g_{\rho} \gamma_{\mu} \tau_{a} \rho^{a\mu} \right] \psi$$

$$+ \frac{1}{2} \partial_{\mu} \sigma \partial^{\mu} \sigma - \frac{1}{2} m_{\sigma}^{2} \sigma^{2} - \frac{1}{3} g_{2} \sigma^{3} - \frac{1}{4} g_{3} \sigma^{4}$$

$$- \frac{1}{4} W_{\mu\nu} W^{\mu\nu} + \frac{1}{2} m_{\omega}^{2} \omega_{\mu} \omega^{\mu} + \frac{1}{4} c_{3} (\omega_{\mu} \omega^{\mu})^{2}$$

$$- \frac{1}{4} R_{\mu\nu}^{a} R^{a\mu\nu} + \frac{1}{2} m_{\rho}^{2} \rho_{\mu}^{a} \rho^{a\mu}.$$

- : nucleons; □: scalar-isoscalar meson
- ! : vector-isoscalar meson; □: vector-isovector meson

$$W_{\mu\nu} = \partial^{\mu}\omega^{\nu} - \partial^{\nu}\omega^{\mu} \qquad R^{a}_{\mu\nu} = \partial^{\mu}\rho^{a\nu} - \partial^{\nu}\rho^{a\mu} + g_{\rho}\epsilon^{abc}\rho^{b\mu}\rho^{c\nu}$$

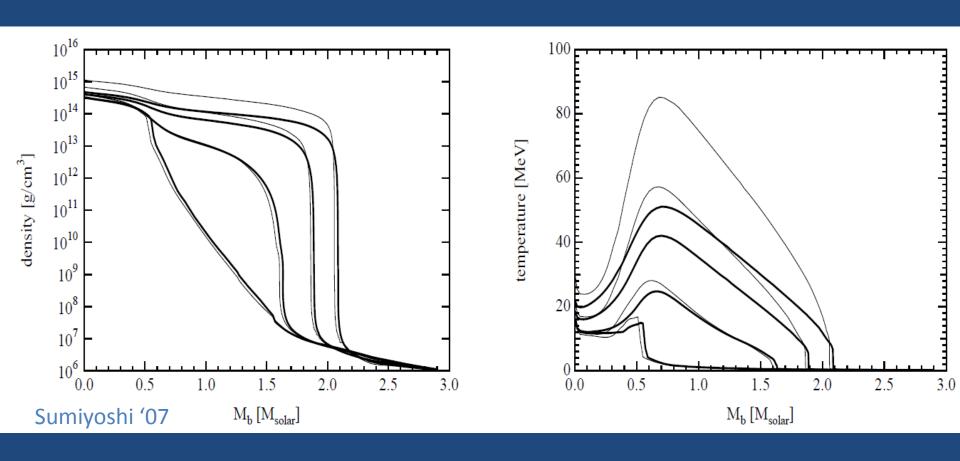
✓ The meson masses and coupling constants are determined to reproduce the properties of nuclear matter at its saturation as well as of finite nuclei.

Comparison of Standard EOS's

	incompressibility K [MeV]	bulk symmetry energy [MeV]	Maximum NS mass [M.]
Lattimer & Swesty's EOS	180	29.3	1.8
	220	29.3	2.0
	375	29.3	2.7
Shen's EOS	281	36.9	2.2
Wol □'s EOS	262	32.9	2.2

- ✓ Shen's EOS has a large symmetry energy.
- ✓ Lattimer & Swesty's EOS with K = 180MeV is too soft although it has been frequently used in the literature.
- ✓ Difference of EOS's manifests itself at later phases. It is more remarkable for black hole formations.

✓ Softer LS EOS gives a more compact and hotter PNS and the BH formation occurs earlier.



- ✓ Other options are highly welcome.
 - relativistic Brueckner-Hartree-Fock approx., variational method, etc.
 - hyperons and Meson condensations
 - quark matter

Neutrinos and Weak Interactions

- ✓ Neutrinos are not in equilibrium with matter in general and their distributions should be somehow solved.
 - Neutrinos can be treated as classical particles.
 - Kinetic descriptions are necessary in principle.

$$p^{-}\frac{@f(x;p)}{@x^{\square}} + \frac{dp^{i}}{d\square}\frac{@f(x;p)}{@p^{i}} = \frac{\Box f(x;p)}{\Box}$$

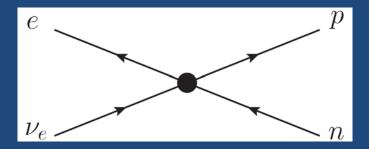
✓ Interactions of v's give the source terms of the Boltzmann eqs. as well as the Euler and Y_e eqs.

$$r \ \ \, T_{M}^{\square} = \square r \ \ \, T_{R}^{\square} = \square \left[\begin{array}{c} \frac{d^{3}p}{E(p)} p^{\square} \end{array} \right] \frac{f(x;p)}{\Box}$$

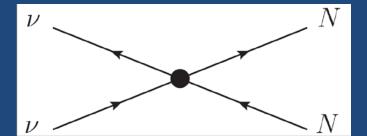
$$r \ \ \, n_{e}^{\square} = \square r \ \ \, n_{e}^{\square} = \square \left[\begin{array}{c} \frac{d^{3}p}{E(p)} p^{\square} \end{array} \right] \frac{f(x;p)}{\Box}$$

Major Reactions

- ✓ The following reactions have large cross sections and are commonly included in simulations.
 - absorptions and emissions on free nucleons
 - reaction rates roughly proportional to ε_{v}^{2}
 - mainly responsible for matter heating below stalled shocks

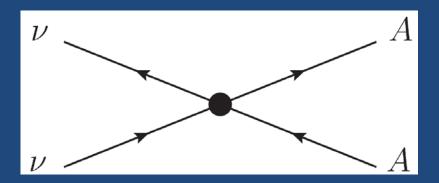


- scatterings on free nucleons
 - reaction rates roughly proportional to ε_v^2
 - nearly iso-energetic



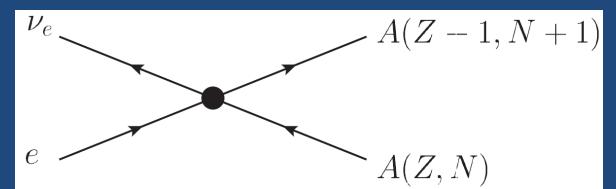
coherent scatterings on nuclei

- reaction rates roughly proportional to ε_{v}^{2} and A^{2}
- mainly responsible for neutrino trapping
- nearly iso-energetic

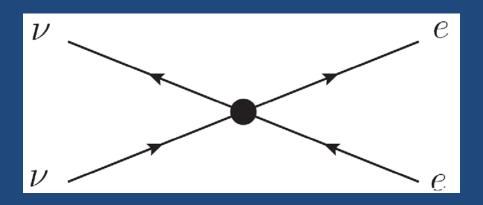


electron captures on Nuclei

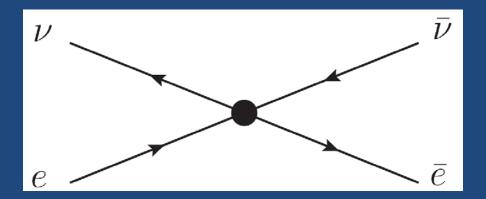
- reaction rates roughly proportional to $\varepsilon_{\rm v}^{2}$
- mainly responsible for Y_e depletion in the collapsing phase



- scatterings on electrons and positrons
 - reaction rates smaller and roughly proportional to $\varepsilon_{\rm v}$
 - thermalizing neutrinos

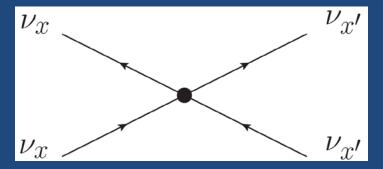


- annihilations and creations of electron and positron pairs
 - reaction rates smaller and comparable to electron scatterings
 - one of main sources of μ and τ neutrinos

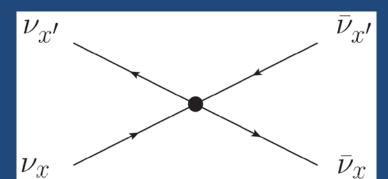


Additional Reactions

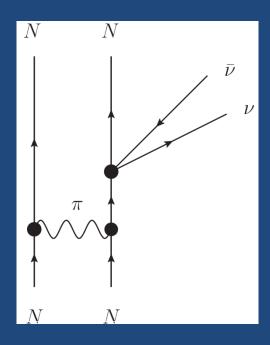
- ✓ The following reactions are as important as electron scatterings and pair processes.
- scatterings on neutrinos
 - reaction rates comparable to electron scatterings
 - important for spectral softening for μ and τ neutrinos



- pair annihilations and creations of neutrinos
 - reaction rates comparable to electron scatterings
 - important for spectral softening for μ and τ neutrinos

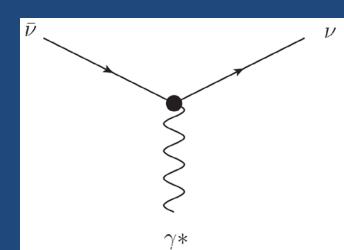


- nucleon bremsstrahlung of neutrino pairs
 - one of main sources of μ and τ neutrinos
 - sometimes greater than pair annihilations of e⁺e⁻



■ plasmon decays

- a source of μ and τ neutrinos
- usually minor



Minor Corrections

- recoils of nucleons
 - Nucleon masses are commonly assumed to be infinity and nucleon recoils are ignored.
- nucleon correlations
 - Nucleons are usually assumed to be free but they are actually correlated spatially and temporarily by nuclear interactions.
- weak magnetism
 - The hadronic currents have tensor component as well as vector and axial vector components.
- corrections to form factors
 - finite momentum transfer
- modifications of phase space by magnetic fields
 - Landau states and magnetic moments

Collision Terms

$$\frac{1}{c} = (p \square u) S$$
 S: reaction rates in the local comoving frame

Emissions and Absorptions

$$S = (R^{e}(p)(1 + f(p)) \square R^{a}(p)f(p)); \quad R^{e}(p) = e^{-|C|} R^{a}(p)$$

Scatterings

$$S = \frac{d^{3}p^{0}}{p_{0}^{0}} (R^{in}(p^{0}; p)f(p^{0})(1 \square f(p)) \square R^{out}(p; p^{0})f(p)(1 \square f(p^{0})));$$

$$R^{in}(p^{0}; p) = e^{\square(E_{\square}^{0} \square E_{\square})} R^{out}(p; p^{0})$$

Pair processes

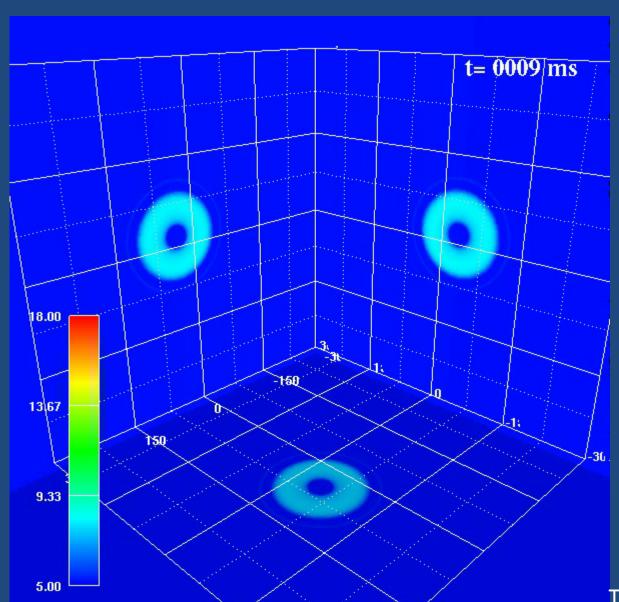
$$S = \frac{d^{3}p^{0}}{p_{0}^{0}} (R^{p}(p; p^{0})(1 \square f(p))(1 \square f(p^{0})) \square R^{a}(p; p^{0})f(p)f(p^{0});$$

$$R^{a}(p; p^{0}) = e^{\square(E_{\square} + E_{\square}^{0})}R^{p}(p; p^{0})$$

Summary

- ✓ To reveal the CCSNe mechanism we need to solve the radiation-(magneto-)hydrodynamics in multi-D with microphysical inputs being properly taken into account.
- ✓ Both numerics and input physics should be improved further.
- ✓ CCSNe and related high energy phenomena will provide us with invaluable information on hadron and neutrino physics.

Latest 3D Simulation



Takiwaki et al '12