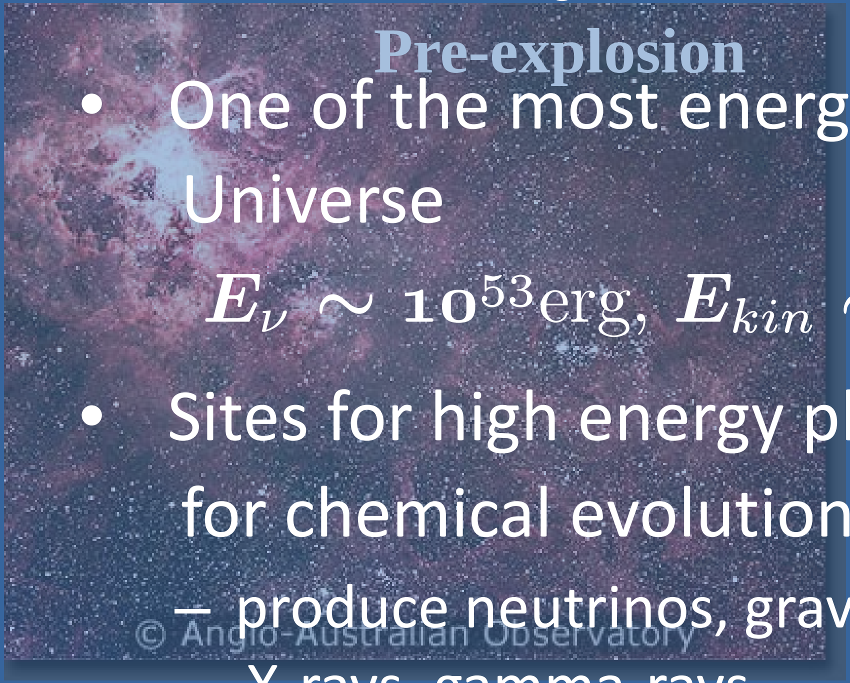
超新星爆発の物理と数値シミュレーション

Shoichi YAMADA

Science & Engineering, Waseda Univ.

Fundamentals of Core-Collapse Supernovae

- Triggered by the gravitational collapse of massive stars ($\gtrsim 10\mathrm{M}_\odot$)



- One of the most energetic phenomena in the Universe

  $$E_\nu \sim 10^{53}\mathrm{erg},\ E_{kin} \sim 10^{51}\mathrm{erg},\ E_\gamma \sim 10^{49}\mathrm{erg}$$

- Sites for high energy phenomena and important for chemical evolutions in the universe
  - produce neutrinos, gravitational waves, cosmic rays, X-rays, gamma-rays
  - nucleosynthesis of heavy elements

Challenges in Supernova Research

Supernova is a complex interplay of

- **Micro Physics**
  - weak interactions
    - neutrino interaction rates with matter
    - neutrino oscillations
  - nuclear physics
    - equation of state
    - many body effects on neutrino reaction rates

- **Macro Physics**
  - hydrodynamics
    - rotation
    - convection
  - radiative transport
  - general relativity
    - gravitational waves
  - magnetic field

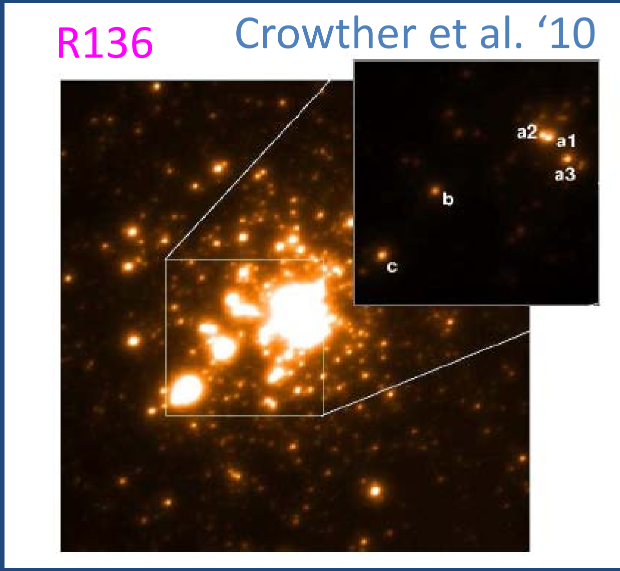
We have to treat them all simultaneously and consistently.

Which Mass Stars Should We Blow Up?

✓ The present universe may be producing stars as massive as ~300M_{solar}.

✓ There is observational evidence that NS's are formed from very massive stars.

- SGR 1806-20: $\gtrsim 50\mathrm{M}_\odot$
- anomalous X-ray pulsar in young massive galactic cluster Westerlund 1: $\gtrsim 40\mathrm{M}_\odot$
- anomalous X-ray pulsar 1E1048.1-5937 embedded in stellar wind bubble: $\sim 30-40\mathrm{M}_\odot$



R136

Crowther et al. '10

✓ The best bet for the minimum mass to produce CCSNe is 8±1M_{solar} at present.

✓ SNeII-P have been observed to be produced by 8.5-17M_{solar} stars.

✓ Most of massive stars may explode to produce neutron stars!

— Core masses are not monotonic owing to mass losses.

Challenges in Supernova Research

The supernova theory must address the following issues :

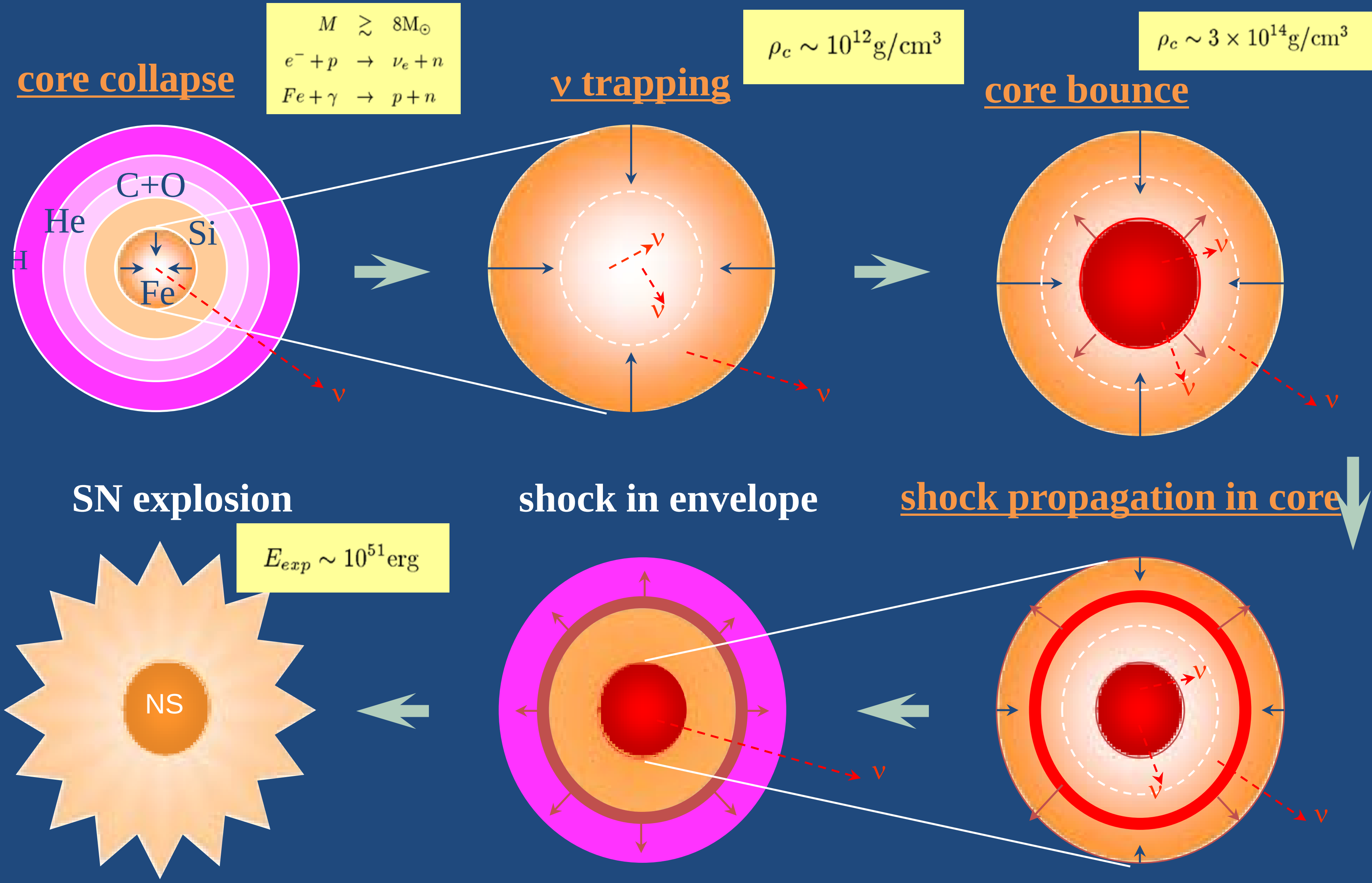
- ✓ How does the explosion occur and are the neutron star mass and explosion energy determined?
- ✓ What are the mass thresholds for NS/BH formations?
- ✓ W
  p

CCSNe can be a new probe into the properties of dense hadronic matter as well as neutrinos!

- ✓ What is the relationship with other high energy objects such as GRBs
  — **hypernovae, magnetars**
- ✓ How do syntheses of heavy elements proceed?
  — **explosive nucleosynthesis, r-process**

Canonical Evolutions of Core Collapse

Scenario of Collapse-Driven Supernovae



core collapse

$$M \gtrsim 8M_\odot$$
$$e^- + p \rightarrow \nu_e + n$$
$$Fe + \gamma \rightarrow p + n$$

ν trapping

$$\rho_c \sim 10^{12}\mathrm{g/cm^3}$$

core bounce

$$\rho_c \sim 3 \times 10^{14}\mathrm{g/cm^3}$$

SN explosion

$$E_{exp} \sim 10^{51}\mathrm{erg}$$

shock in envelope

shock propagation in core

Summary of CCSNe

✓ CCSN is an explosion of massive stars triggered by core collapse.

✓The liberated gravitational energy ~10^{53}erg is far greater

We are required to compute gas dynamics with the energy-transport by neutrinos and thermodynamical nature of matter being taken into account properly.

✓ ν's are the only agents of non-local energy transport that can tap the internal energy stored in PNS.

✓ The density changes from ~10^{10} to 3-5 x 10^{14} g/cm^3 and temperature varies from ~10^{10} to a few x 10^{11}K.

✓ The matter changes its nature drastically at nuclear saturation density ~3×10^{14}g/cm^3.

Basic Equations & Input Physics

ν-Radiation (Magneto) Hydrodynamics

✓ Gas dynamics is described with (magneto-) hydrodynamical equations.

— Dissipations can be neglected, since particle mean free paths are very short.

$$\lambda \sim 10^{-10}\mathrm{cm}\left(\frac{\sigma}{10^{-24}\mathrm{cm}^2}\right)\left(\frac{\rho}{10^{10}\mathrm{g}/\mathrm{cm}^3}\right)$$

— Variables to be solved are baryonic number density, n_B, electron fraction, Y_e, (= proton fraction, Y_p), entropy per baryon, s, (alternatively, temperature, T, or internal energy density, e, total energy density, E, etc.) and velocities, $\boldsymbol{v}$, (plus magnetic fields, $\boldsymbol{B}$).

— Continuity eq., Euler eqs., eq. for electron fraction (plus induction eq.) are solved, employing an appropriate EOS.

— Spacetime geometry (gravitational potential in Newtonian gravity) is solved simultaneously.

✓ Continuity Eq.

$$\nabla_\mu (n_B u^\mu) = 0$$: Baryon number conservation

✓ Eq. for Y_e

$$\nabla_\mu (n_L u^\mu) = 0$$: Lepton number conservation

⇨ $$\nabla_\mu (n_e u^\mu) = -\nabla_\mu (n_{\nu_e} u^\mu)$$

⇨ $$u^\mu \nabla_\mu Y_e = -u^\mu \nabla_\mu Y_{\nu_e}$$

↑

Expressed by collision terms of Boltzmann eq.

※ Heavy leptons are not abundant in supernova cores.

Energy-momentum tensors

$$T^{\mu\nu} = T^{\mu\nu}_{M} + T^{\mu\nu}_{R} (+ T^{\mu\nu}_{EM})$$

$$\begin{aligned}
T^{\mu\nu}_{M} &= \rho u^{\mu} u^{\nu} + (g^{\mu\nu} + u^{\mu} u^{\nu}) p \\
T^{\mu\nu}_{R} &= E_R u^{\mu} u^{\nu} + u^{\mu} F^{\nu}_{R} + F^{\mu} u^{\nu}_{R} + P^{\mu\nu}_{R} \\
\Bigg(T^{\mu\nu}_{EM} &= \frac{1}{4\pi} \left[F^{\mu\rho} F^{\nu}_{\rho} - \frac{1}{4} g^{\mu\nu} F^{\rho\sigma} F_{\rho\sigma} \right] \Bigg)
\end{aligned}$$

✓ Euler Equations

$$\nabla_{\nu} T^{\mu\nu} = 0$$

$$\text{or } \nabla_{\nu} T^{\mu\nu}_{M} = -\underset{\uparrow}{\nabla_{\nu} T^{\mu\nu}_{R}} (-\nabla_{\nu} T^{\mu\nu}_{EM})$$

Expressed by collision terms of Boltzmann eq.

- ✓ Einstein eq.

$$G^{\mu\nu} = T^{\mu\nu}$$

- ✓ Equation of State (EOS)

$$p = p(n_B, \varepsilon, Y_e)$$

※ ε can be replaced by other thermodynaical variable such as *s*, *T*, etc.

- ✓ Induction eq.

$$\begin{aligned} dF^{\mu\nu} &= 0 \\ \nabla_\nu F^{\nu\mu} &= \frac{4\pi}{c} j^\mu \\ F^{\mu\nu} u_\nu &= 0 \end{aligned}$$

✓ The agents of radiative transport of energy and momentum are neutrinos.

- — Wave lengths of neutrinos are much shorter than the hydrodynamical length scale and neutrinos can be treated as particles and be described by kinetic equations such as Boltzmann equation.
  - Neutrino oscillations are the only processes, in which wave characters of neutrinos manifest themselves in macroscopic phenomena.
- — Mean free paths of neutrinos are longer than the hydrodynamic length scale at low densities ($\rho_B \lesssim 10^{11} \mathrm{g/cm}^3$).
- — Neutrino distributions are not the Fermi-Dirac distributions even locally at low densities and should be solved with the kinetic equations.

- ✓ Only electron-type neutrinos are produced before core bounce but all six types of neutrinos are abundant after bounce.
- ✓ Unless heavy leptons are produced, there is no difference between μ- and τ-neutrinos.
- ✓ The distribution of ν_μ (ν_τ) is different from that of $\bar{\nu}_\mu$ ($\bar{\nu}_\tau$) in principle. The difference is minor and neglected in practice.
- ✓ Tiny neutrino masses are neglected unless neutrino oscillations are considered.

$$m_\nu \lesssim 1\mathrm{ev} \ll E_\nu \sim O(\mathrm{MeV})$$

✓ Boltzmann Eq.

$$\frac{df}{d\lambda} = \left(\frac{\delta f}{\delta\lambda}\right)_c \Longrightarrow \quad p^\mu \frac{\partial f(x,p)}{\partial x^\mu} + \frac{dp^i}{d\lambda}\frac{\partial f(x,p)}{\partial p^i} = \left(\frac{\delta f(x,p)}{\delta\lambda}\right)_c$$

$$\lambda : \text{affine parameter} \quad p^\mu = \frac{dx^\mu}{d\lambda}, \quad \frac{dp^\mu}{d\lambda} = \Gamma^\mu_{\rho\sigma} p^\rho p^\sigma : \text{geodesic eq.}$$

Number current & Energy-momentum Tensor

$$n^\mu = \int \frac{d^3p}{E(p)}\, p^\mu f(x,p), \quad T^{\mu\nu} = \int \frac{d^3p}{E(p)}\, p^\mu p^\nu f(x,p)$$

※ In an orthonormal frame

Change of Number & Energy-momentum Densities

$$\nabla_\mu n^\mu = \int \frac{d^3p}{E(p)} \left(\frac{\delta f(x,p)}{\delta\lambda}\right)_c, \quad \nabla_\nu T^{\mu\nu} = \int \frac{d^3p}{E(p)}\, p^\mu \left(\frac{\delta f(x,p)}{\delta\lambda}\right)_c$$

※ In an orthonormal frame

Microphysical inputs in core collapse simulations:

- EOS : various thermodynamical quantities, such as p, T, μ, c_s and nuclear abundance X_A, as functions of 3 independent variables of your choice, e.g. (n_B, ε, Y_e)

- ν interactions :
  plugged in the collision term of Boltzmann eqs.

$$\left(\frac{\delta f(x,p)}{\delta \lambda}\right)_c$$

EOS

Lattimer & Swesty's EOS

✓ Based on a model free energy per baryon with the Skyrme-type parametrization:

$$f(n, Y_p, T) = E(n, Y_p, T)/n - T s(n, Y_p, T)$$

$$\begin{aligned} E(n, Y_p, T) &= \sum_t \frac{\hbar^2 \tau_t}{2\, m_t^*} + [\, a + 4b\, Y_p (1 - Y_p)\,]\, n^2 + c\, n^{1+\delta} - Y_p n\, \Delta, \\ s(n, Y_p, T) &= \sum_t \left(\frac{5\, \hbar^2 \tau_t}{6\, m_t^*\, T} - n_t\, \eta_t \right) / \, n. \end{aligned}$$

t : isospin, τ_t : kinetic energy density, m_t^* : effective mass
Δ : n-p mass difference, $V_t = \delta E / \delta n_t$, $\eta_t = (\mu_t - V_t)/k_B T$

✓ The parameters *a*, *b*, *c* and δ are determined by the properties of zero temperature symmetric nuclear matter at its saturation density: saturation density, binding energy, bulk symmetry energy and bulk incompressibility.

Shen's EOS

- ✓ Relativistic mean field theory
- ✓ Nuclear interactions are described by meson exchanges.

$$
\begin{aligned}
\mathcal{L}_{RMF} \;=\;& \bar{\psi}\left[i\gamma_\mu\partial^\mu - M - g_\sigma\sigma - g_\omega\gamma_\mu\omega^\mu - g_\rho\gamma_\mu\tau_a\rho^{a\mu}\right]\psi \\
& +\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 \\
& -\frac{1}{4}W_{\mu\nu}W^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{4}c_3\left(\omega_\mu\omega^\mu\right)^2 \\
& -\frac{1}{4}R^a_{\mu\nu}R^{a\mu\nu} + \frac{1}{2}m_\rho^2\rho^a_\mu\rho^{a\mu}.
\end{aligned}
$$

ψ : nucleons, σ : scalar-isoscalar meson

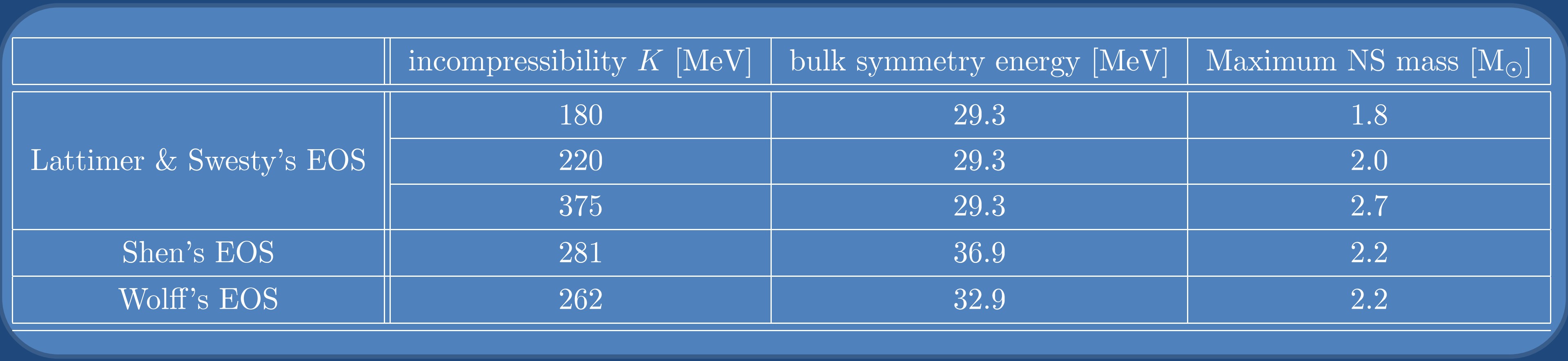
ω : vector-isoscalar meson, ρ : vector-isovector meson

$$W_{\mu\nu} = \partial^\mu\omega^\nu - \partial^\nu\omega^\mu$$

$$R^a_{\mu\nu} = \partial^\mu\rho^{a\nu} - \partial^\nu\rho^{a\mu} + g_\rho\epsilon^{abc}\rho^{b\mu}\rho^{c\nu}$$

- ✓ The meson masses and coupling constants are determined to reproduce the properties of nuclear matter at its saturation as well as of finite nuclei.

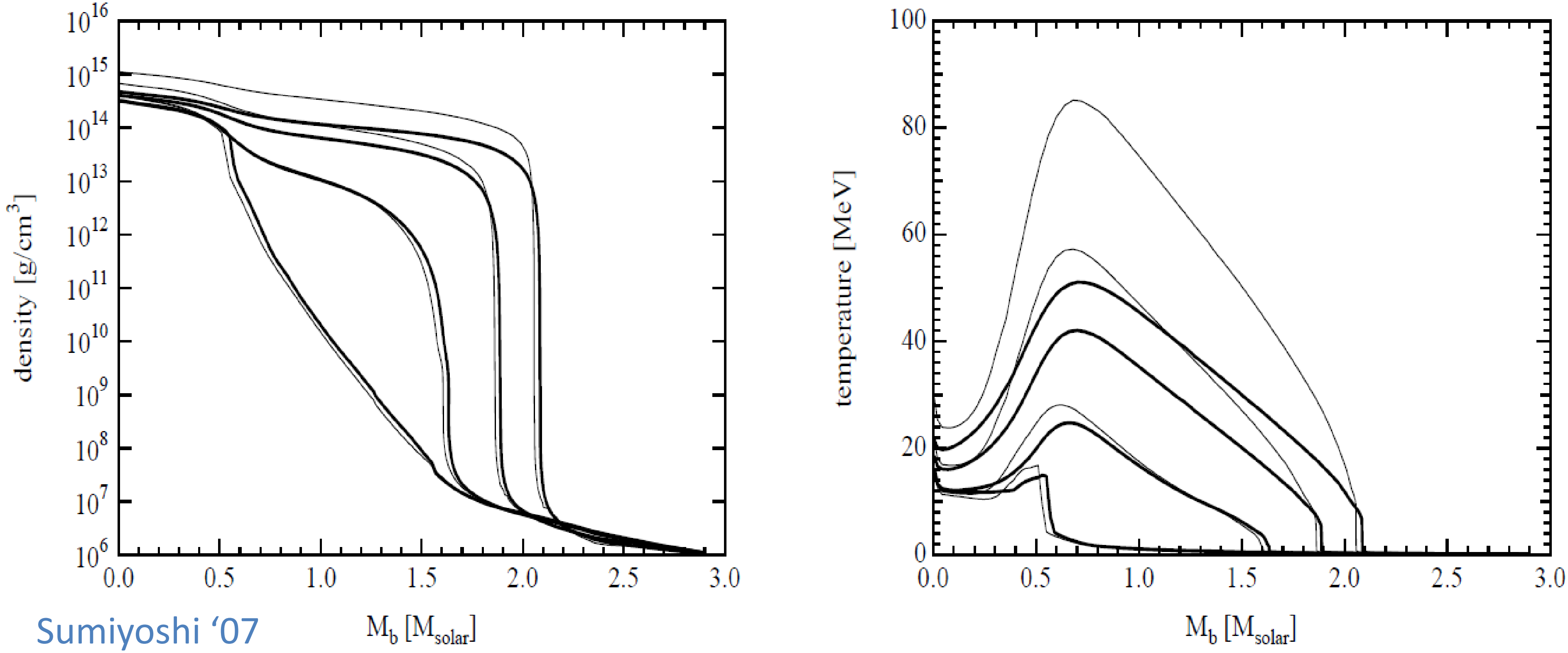
Comparison of Standard EOS’s



	incompressibility K [MeV]	bulk symmetry energy [MeV]	Maximum NS mass [$M_\odot$]
Lattimer & Swesty’s EOS	180	29.3	1.8
	220	29.3	2.0
	375	29.3	2.7
Shen’s EOS	281	36.9	2.2
Wolff’s EOS	262	32.9	2.2

- ✓ Shen’s EOS has a large symmetry energy.
- ✓ Lattimer & Swesty’s EOS with K = 180MeV is too soft although it has been frequently used in the literature.
- ✓ Difference of EOS’s manifests itself at later phases. It is more remarkable for black hole formations.

✓ Softer LS EOS gives a more compact and hotter PNS and the BH formation occurs earlier.



Sumiyoshi ‘07

✓ Other options are highly welcome.

— relativistic Brueckner-Hartree-Fock approx., variational method, etc.
— hyperons and Meson condensations
— quark matter

Neutrinos and Weak Interactions

✓ Neutrinos are not in equilibrium with matter in general and their distributions should be somehow solved.

- Neutrinos can be treated as classical particles.
- Kinetic descriptions are necessary in principle.

$$p^\mu \frac{\partial f(x,p)}{\partial x^\mu} + \frac{dp^i}{d\lambda}\frac{\partial f(x,p)}{\partial p^i} = \underline{\left(\frac{\delta f(x,p)}{\delta \lambda}\right)_c}$$

✓ Interactions of ν's give the source terms of the Boltzmann eqs. as well as the Euler and Y_e eqs.

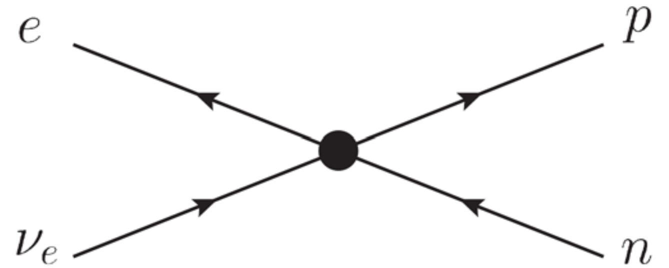
$$\nabla_\nu T_M^{\mu\nu} = -\nabla_\nu T_R^{\mu\nu} = \underline{-\int \frac{d^3p}{E(p)}\, p^\mu \left(\frac{\delta f(x,p)}{\delta \lambda}\right)_c}$$

$$\nabla_\mu n_e^\mu = -\nabla_\mu n_{\nu_e}^\mu = \underline{-\int \frac{d^3p}{E(p)}\, p^\mu \left(\frac{\delta f(x,p)}{\delta \lambda}\right)_c}$$

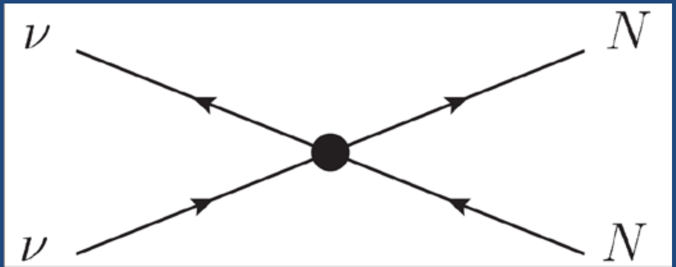
Major Reactions

✓ The following reactions have large cross sections and are commonly included in simulations.

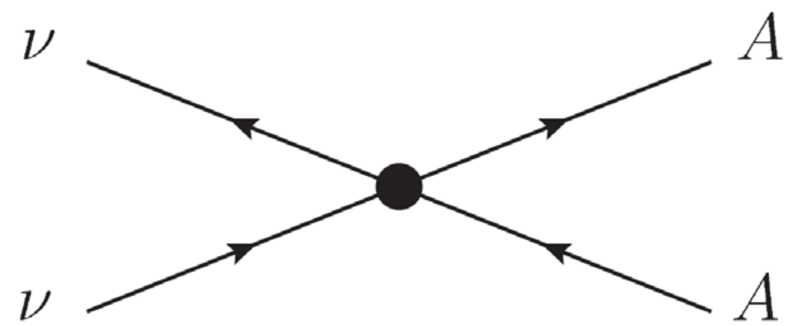
- absorptions and emissions on free nucleons
  - — reaction rates roughly proportional to ε_ν^2
  - — mainly responsible for matter heating below stalled shocks



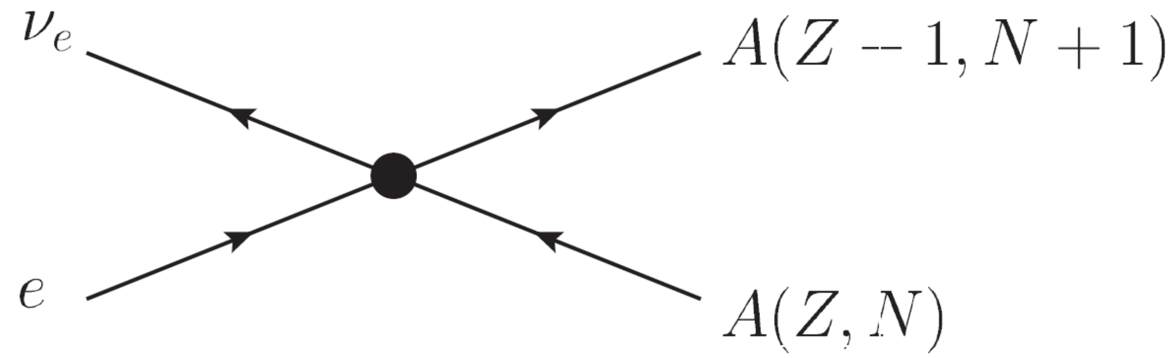
- scatterings on free nucleons
  - — reaction rates roughly proportional to ε_ν^2
  - — nearly iso-energetic



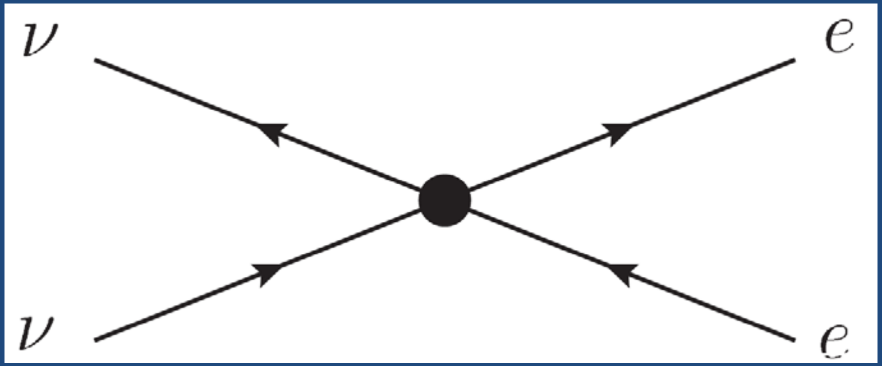
- coherent scatterings on nuclei
  - — reaction rates roughly proportional to ε_ν^2 and A^2
  - — mainly responsible for neutrino trapping
  - — nearly iso-energetic



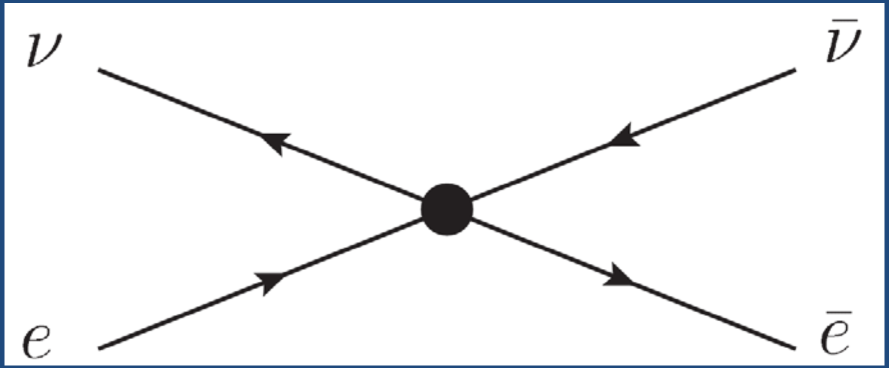
- electron captures on Nuclei
  - — reaction rates roughly proportional to ε_ν^2
  - — mainly responsible for Y_e depletion in the collapsing phase



- scatterings on electrons and positrons
  - — reaction rates smaller and roughly proportional to ε_ν
  - — thermalizing neutrinos



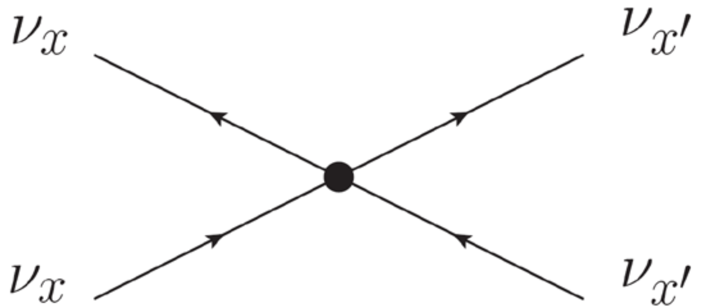
- annihilations and creations of electron and positron pairs
  - — reaction rates smaller and comparable to electron scatterings
  - — one of main sources of μ and τ neutrinos



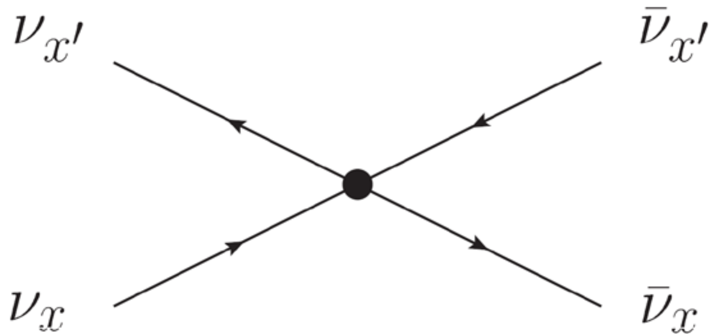
Additional Reactions

✓ The following reactions are as important as electron scatterings and pair processes.

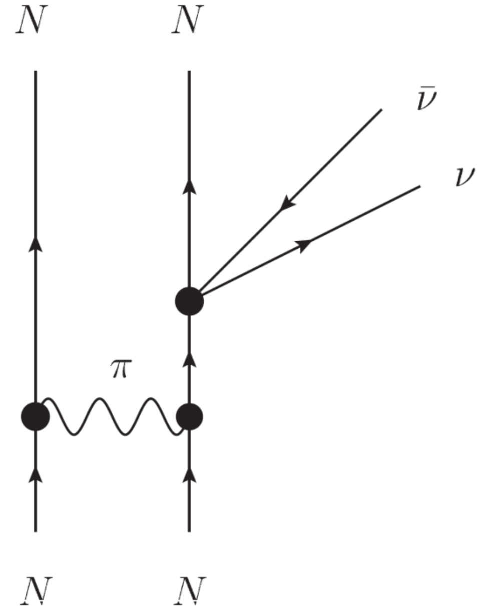
- scatterings on neutrinos
  - — reaction rates comparable to electron scatterings
  - — important for spectral softening for μ and τ neutrinos



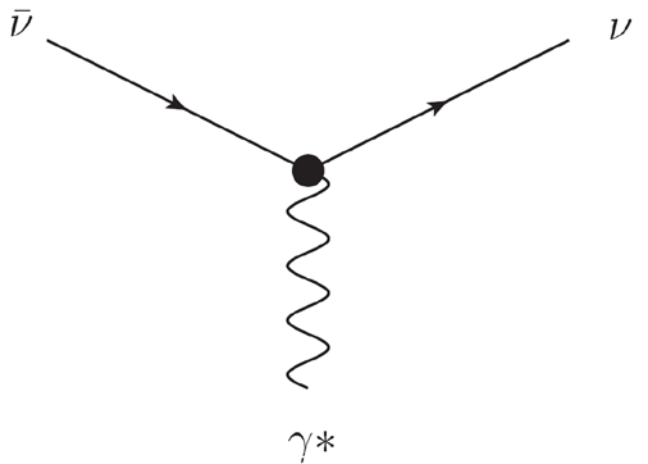
- pair annihilations and creations of neutrinos
  - — reaction rates comparable to electron scatterings
  - — important for spectral softening for μ and τ neutrinos



- nucleon bremsstrahlung of neutrino pairs
  - — one of main sources of μ and τ neutrinos
  - — sometimes greater than pair annihilations of e^+e^-



- plasmon decays
  - — a source of μ and τ neutrinos
  - — usually minor



Minor Corrections

- recoils of nucleons
  - Nucleon masses are commonly assumed to be infinity and nucleon recoils are ignored.
- nucleon correlations
  - Nucleons are usually assumed to be free but they are actually correlated spatially and temporarily by nuclear interactions.
- weak magnetism
  - The hadronic currents have tensor component as well as vector and axial vector components.
- corrections to form factors
  - finite momentum transfer
- modifications of phase space by magnetic fields
  - Landau states and magnetic moments

etc.

Collision Terms

$$\left(\frac{\delta f}{\delta \lambda}\right)_c = (p \cdot u)\, S \qquad S\text{: reaction rates in the local comoving frame}$$

- Emissions and Absorptions

$$S = (R^e(p)(1 - f(p)) - R^a(p) f(p)), \qquad R^e(p) = e^{-\beta(E_\nu - \mu_\nu)} R^a(p)$$

- Scatterings

$$S = \int \frac{d^3 p'}{p'_0} (R^{in}(p', p) f(p')(1 - f(p)) - R^{out}(p, p') f(p)(1 - f(p'))),$$

$$R^{in}(p', p) = e^{\beta(E'_\nu - E_\nu)} R^{out}(p, p')$$

- Pair processes

$$S = \int \frac{d^3 p'}{p'_0} (R^p(p, p')(1 - f(p))(1 - f(p')) - R^a(p, p') f(p) f(p')),$$

$$R^a(p, p') = e^{\beta(E_\nu + E'_\nu)} R^p(p, p')$$

Summary

- ✓ To reveal the CCSNe mechanism we need to solve the radiation-(magneto-)hydrodynamics in multi-D with microphysical inputs being properly taken into account.

- ✓ Both numerics and input physics should be improved further.

- ✓ CCSNe and related high energy phenomena will provide us with invaluable information on hadron and neutrino physics.